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Dise, Robert L.; Caulk, Robert F.

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ROBERT L. DISE
AND
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by

Robert L. Dise

Lieutenant Commander, United States Navy

and

Robert F. Caulk

Lieutenant, United States Navy

Submitted in partial fulfillment of
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MASTER OF SCIENCE

IN

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Abstract

In modern technology accuracy requirements have become more important in nearly every field of measurement. Position encoders, used to convert such items as shaft rotation into discreet units of measure, are one method developed to improve accuracy.

The Servo Section of the Philco Corporation, Western Development Laboratories, Palo Alto, California has designed and worked on servo control systems employing disc-type position encoders for accurately controlling the position of rotating shafts, such as on antennas. Two analog-to-digital encoders, both capable of reading to 2^{16} (1 part in 65,536) positions, were under consideration by the laboratory. One of these, the Model A9SP16 manufactured by the Baldwin Piano Company, was a single disc employing an optical readout principle. The other, produced by the Datex Division of the G. M. Giannini & Company, Inc., used two discs and a brush readout arrangement. An analysis, as well as further testing, of two similar servo systems employing these two encoders was desired and was undertaken by the authors during the period June - July 1959.

The authors wish to acknowledge the help and friendship accorded them by the engineers and technicians of the Servo Section especially Mr. Lynn J. Harvey and Mr. Joseph L. Heim. Sincere appreciation is also extended to Dr. Gene F. Franklin of Stanford University, a Philco consultant, and to Professor Charles H. Rothauge, Dr. Eng., faculty adviser.

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List of Symbols

Symbol	Meaning
K_e	Voltage speed coefficient for the motor transfer function calculation
K_v	Motor Gain constant
τ_m	Time constant for the motor, seconds
J	Motor inertia, oz-in^2
K_c	Comparator Gain constant in linear region, volts per radian
K_a	Amplifier Gain constant, volt per volt
K_T	Tachometer Gain constant, volt per radian per second
K_{eg}	Equivalent or overall gain constant, volt per radian per second
f_s	Sampling frequency, pulses per second
f_L	Limiting frequency for stability, pulses per second
T	Sampling period, seconds between pulses
$\dot{\theta}_m$	Motor speed, radians per second
E(t)	Error as a function of time, radians
θ_o	Output of Encoder shaft, radians
θ_i	Input from A register, radians
e_{ss}	Steady state error, radians
V_T	dc tachometer voltage before chopping, volts

1. INTRODUCTION

With the ever-increasing amount of technical data and information needing reduction into a usable form, with the ever-increasing demands on single systems for multiple uses, with the ever-increasing demands for greater accuracy in data and in systems control the development of analog-to-digital[1] converters was as natural and necessary a step as that of a-c following d-c. By digitizing the data it can be stored for any length of time, transmitted over any distance and re-transmitted, and detected or read as many times as necessary with no loss in accuracy. Digital signals are relatively immune to distortion by noise and nonlinearities. Analog signals are distorted by each of these processes. Not only that but certain types of information can be measured to much higher accuracies by digital means. Analog-to-digital converters or encoders are actually devices for obtaining the digital representation of some item such as a linear displacement, an angular rotation, a frequency (by counting over an accurate time interval), an acceleration, etc. They are easily integrated into servo systems, in fact are well suited for sampled data systems by the very nature of this output. The type of interest here are those that convert mechanical shaft rotation to digital information using an electrical voltage. The mechanical motion is subdivided into incremental bits by the encoder disc, and the conversion to digital form is

[1]"Analog" as used in this paper refers to the representation of some independent variable by a dependent variable through a known scaling factor. Analog functions have an infinite number of values since they are continuous functions.

"Digit" refers to the absence or presence of a function, a definite sharp discontinuity.

accomplished by determining the number of increments contained in a particular motion. Instantaneous position can be determined by one of two methods. One type is to have a known zero position and to accumulate a count of the number of increments through which rotation has taken place. The other method is to use a coded pattern on the disc so that any position can be read directly. This latter method applies to both encoders used in the applications in this work. Coding has a further advantage in that it improves the signal-to-noise ratio by reducing the time required to transmit a signal of any given precision.

Coded digital signals can be either serial (pulses counted sequentially) or parallel (pulses counted simultaneously). Serial systems use less transmission equipment, since all data is carried by a single channel, but bandwidth requirements can make the equipment expensive, especially for high information rates. Parallel systems use a separate, narrow-band channel for each code bit but are more expensive for low information rates.

Of the many physical principles that can be used to give automatic reading the four currently used are electrical contacts (commutators and brushes), patterns which control the transmission or reflection of optical energy, magnetic devices and electrostatic means. Each has its own peculiarities, advantages, and limitations. All four of these can be used in coded disc angular encoders, i.e. "converters that read".

The shaft position encoder has its digital scale directly attached to the shaft, permanently referenced and encoded. In the optical and brush systems fixing the position of the readout brushes or the photocells makes a permanent reference. It is only necessary to close the circuits in order to get a coded representation of the analog shaft position. The reading

speeds are limited only by the electronic circuitry connected with the photocells or brushes. Actually, the speed at which samples can be taken depends on the system in which the encoder is used.

The disc encoders are available in single or multiple disc types, with individual discs ranging in size from one to $16\frac{1}{2}$ inches in diameter.

The basic requirements for selecting any one kind of encoder, once it is decided that one is desirable are:

1. It must satisfactorily accept the input to be measured,
2. It must produce an output in the desired code and this output must
   a. Be accurate enough
   b. Have a high enough maximum sampling rate.

The most common means of translating shaft angle into digital code in use today are brush contact or optical. In these the discs have concentric annular rings laid out starting at the periphery and working inward. Each ring represents a binary digit. The least significant digit ring is at the periphery with more significant rings displaced toward the center. The most significant digit is represented by the innermost ring.

Each ring is divided into segments. The segments are left clear or made opaque if optical readout is intended, or made insulating or conducting for brush readout. Fig. 1 shows a typical disc. The angular span of the individual segments of the ring is determined by the significance of the digit. For the least significant digit

$$\frac{360^{\circ}}{2N} = \text{segment span in degrees (called a "bit" or "quantum")}$$

where N is the number of binary digits expressed by the disc. The most

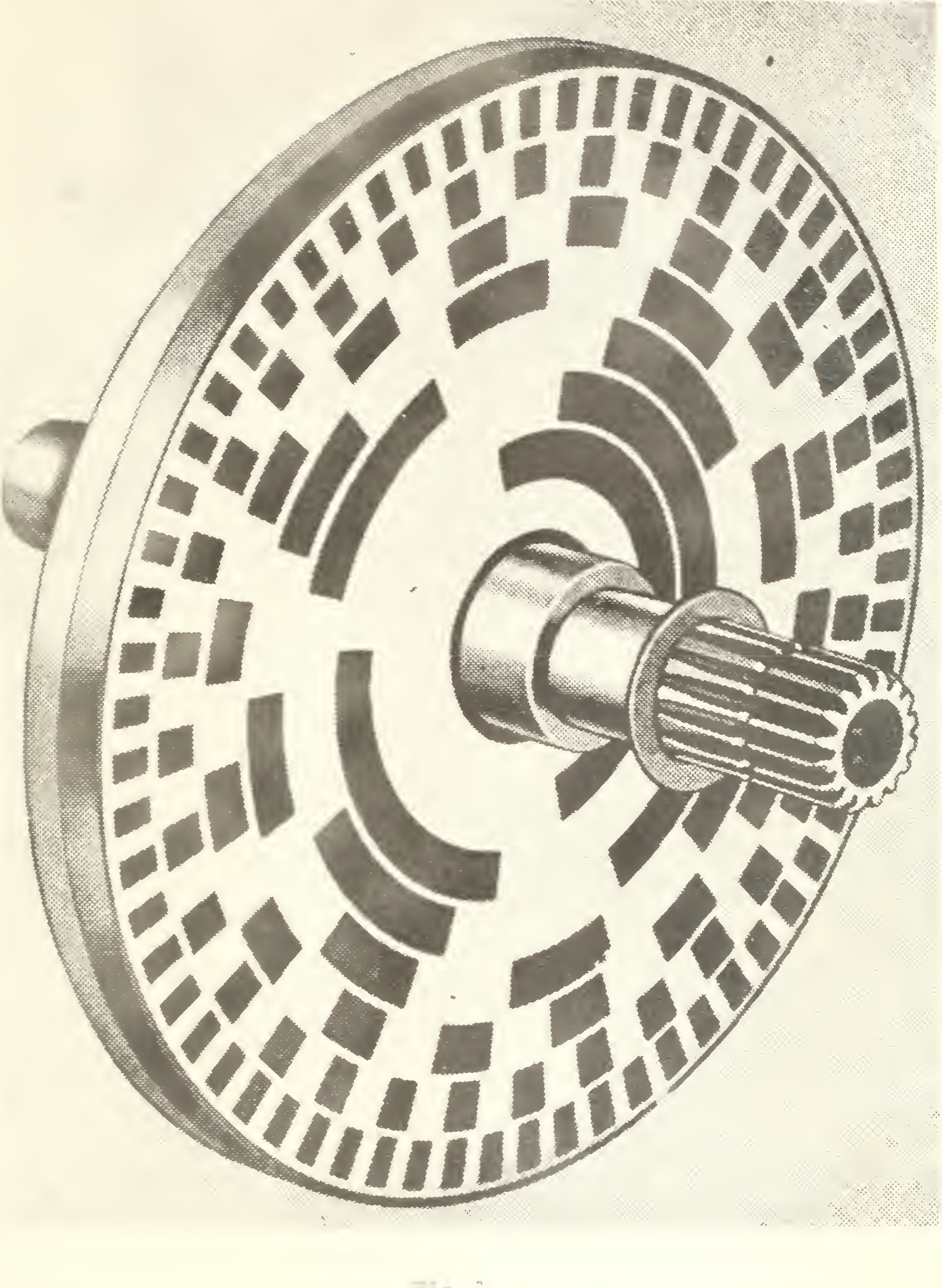


Fig. 1

Typical Binary-coded Disc

significant digit ring contains 180° segments. Successive rings in the direction of decreasing significance contain segments whose lengths are successively half as long as preceeding rings. Segments are alternately conducting and non-conducting (or transparent and translucent) corresponding most commonly to binary one and zero respectively. Binary read-off is used since it is by far the easiest to obtain, it being simply either a "yes" or a "no" situation. (See Appendix A for a detailed description of binary numbers and codes.)

To illustrate the method by which discs are read the brush system will be considered keeping in mind that the discs using photocells are read in the same manner. If the brushes were ideal, i.e. of zero width, and arranged along a radial line, one for each ring, with an extra one called a common, each brush would give an output dependent upon the state (conducting or non-conducting) of the segment that it contacts. The output of the brushes would be a binary representation of the disc position from a fixed reference, the location of the radial line of brushes. The binary representation would be direct reading, and as the shaft (and disc) rotated true binary position could be continuously read.

However photocells and light sources for optical readout, and brushes for electrical readout, are of finite size. If the disc is located so that the readout devices are positioned at the center of a given sector there is no problem but since the discs rotate, ambiguities will arise for the patterns inscribed on the discs may have inaccuracies and concentricity errors may occur between disc and shaft. Since the limits of mechanical construction make it impossible for the pick-up brushes to change contact position simultaneously, there occurs narrow intervals of partial transi-

tion where some, but not all, brushes have made the next contact. This gives rise to ambiguities, particularly if a conventional binary code is used, since a given sector may be mistaken for one far away from it, yielding a large error. This is possible since in a conventional binary code it frequently happens that more than one binary digit will change in going from one number to the next.

There are two methods to correct for this that are in general use at present. One of these is the development of special binary codes which change only one digit at a time, such as the Gray or cyclic code (see Appendix A). The principal disadvantage of the Gray code is the need to reconvert to natural binary or some other convenient code before use in digital equipment.

The second solution is sometimes called "V scanning" due to the "V" pattern of the physical arrangement of the brushes. V scanning calls for two brushes per track, spaced a maximum of one half segment apart, on all rings but the least significant. One of the two brushes is selected to be read through the operation of logical circuitry. In linear converters, V scanning never allows a selected brush any closer to the edge of a segment than 25% of the segment length. In this method the brushes provide an error free reading of a given ring when they are "controlled" by a brush from the adjacent ring in the least significant direction. Each ring in turn is controlled by the next least significant, and the ultimate controlling element is the least significant brush. The rules for brush selection are (see Fig. 2):

1. When the controlling brush (B_0) changes from 1 to 0 select the leading brush (B_1) in the next track.

2. When the controlling brush changes from 0 to 1 select the lagging (B_2) brush in the next track.

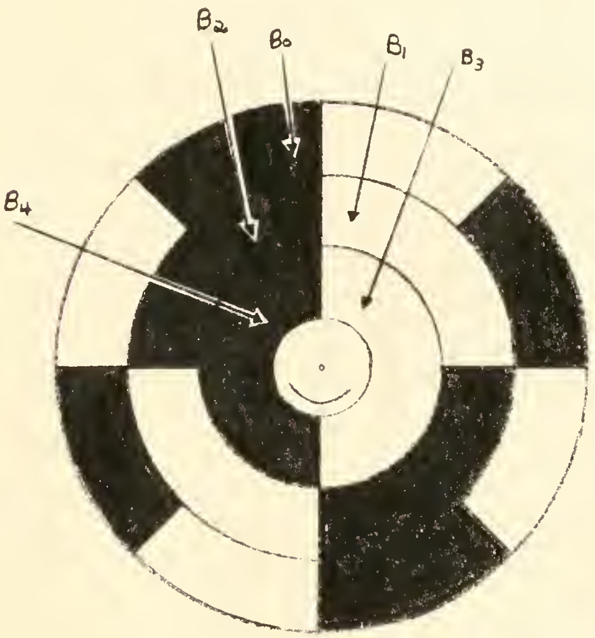


Fig. 2

Simplified Diagram for Description of V-scan

A third and newer type of disc, for use optically or by brush pick-off, that has indications of becoming increasingly valuable is the family of sine-cosine encoders. In angular measurement, high-accuracy encoder applications such as tracking radars and theodolites, they should be of particular value. For instance, when reconstructing a missile's path from radar or theodolite data the sine and cosine functions of the measured angle are of primary interest. In this type instead of coding the tracks in even increments of angular measurement they are coded in the sine-cosine value of the angle. The newer models of these discs have proven just as accurate as the linear coded ones.

In the brush system perhaps the most serious ambiguity problem arises as a result of variations in brush alignment. Brushes are susceptible to vibration and acceleration forces, which can cause intermittent contacts and shifts in brush position. Since shaft friction, resistance to mechanical disturbance, and operating life depend on the brush pressure, a compromise must be made among them in the coder design. Resistance to mechanical forces can in general only be obtained at the expense of increased friction and, therefore, a shorter operating life. Also, due to difficulty in getting sufficient built-in accuracy in manufacturing, brush assembly adjusting screws are needed.

Most of these problems are eliminated in an optical disc with a photoelectric reading method. As mentioned previously, the optical type has a pattern of opaque and transparent areas that can be read by placing the disc between a light source and a photo cell array. An optical slit establishes the width of the "brush" in optical readings. Fig. 3 illustrates one such arrangement. Since there are no rubbing contacts, there is no wear problem and no limit on the disc life or disc operating speed. The photocell assembly and optical slit can be made more rigid than the brushes of a commutator coder, and slits can be made extremely straight and of constant width. The problem of relative slit alignment between rings is therefore not present in optical coders. The reduced dimensions of the segments does make the tolerances on slit alignment very small.

Of course, along with the advantage of optical coders with respect to reading resolution there are also disadvantages, some peculiar to optical systems. For one thing, because of the finite width of the slit and optical diffraction effects, the cell output does not change abruptly

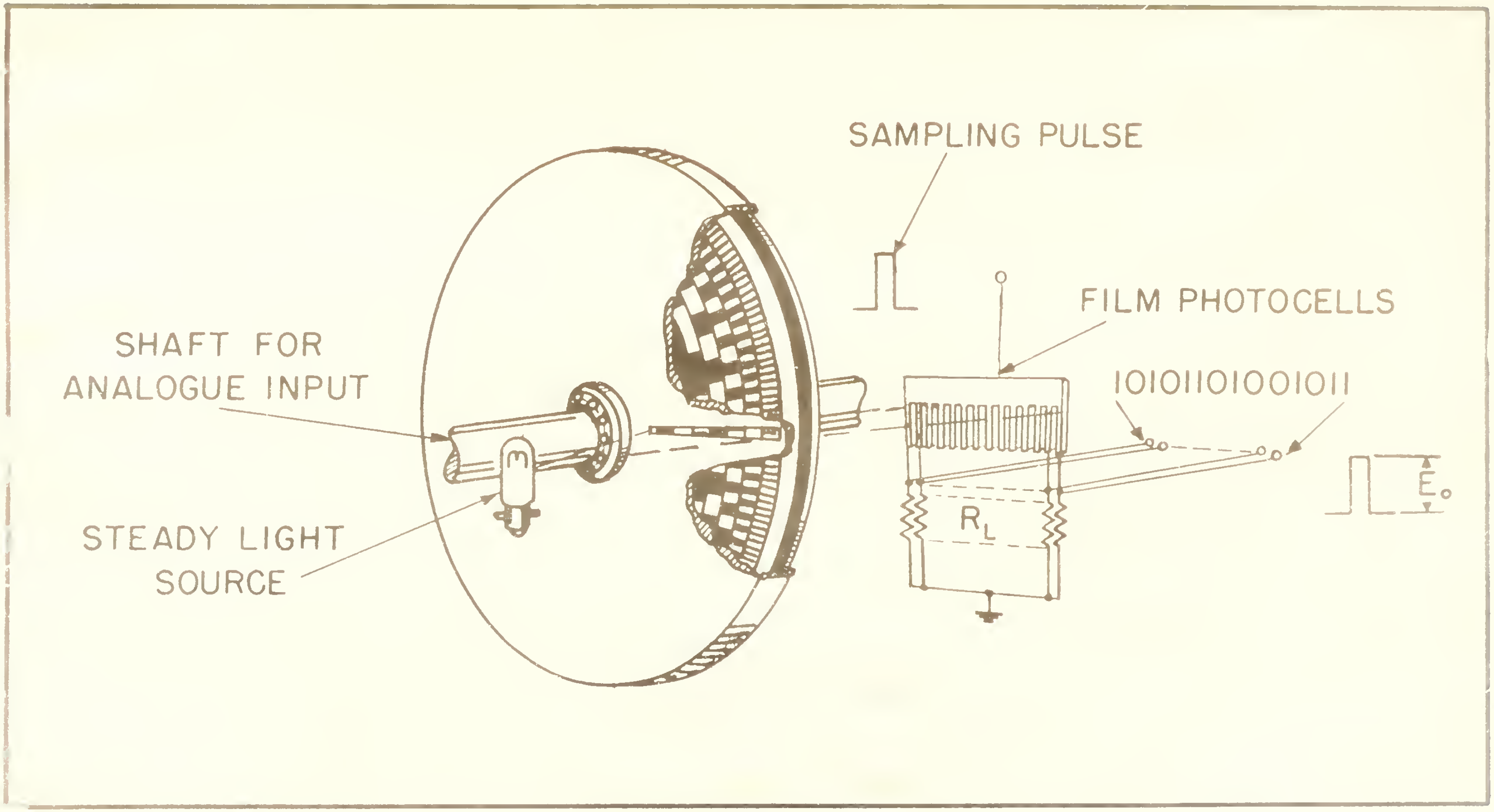


Fig. 3

Optical Reading System

from "dark" to "light" as the disc moves. To detect the exact position of a transition in the pattern, an external amplitude discriminator is required at the cell output.

The optical reading system is more complicated then a commutator and brush system and is thus perhaps just as likely to give troubles. Discs may be read either by flashing the light or by using a steady light and turning the photocells on and off. If a flash lamp is used, the coder requires a high-voltage power supply which may give trouble. Also, cell outputs with flash excitation are so small that an amplifier is required in each cell output, and gain stability is required to maintain a small discrimination band.

The reading system for an optical coder consists of: (1) a light source (2) an optical system, which forms an index beam and, (3) a detector, which is used to determine the index position on a coded plate. Either the index beam or the coded plate may be arbitrarily fixed and the other moved in accordance with the measured parameter.

Several basic optical systems can be used, such as the point energy source illuminating a cylindrical lens or the line source (Fig. 3). In the latter, the one used in the Baldwin encoder, a line source is used to irradiate the coded surface. An optical slit is placed between the coded surface and the detector. This slit forms and determines the dimension of the index, the light radiating from the source in directions other than the straight path to the slit being prevented from reaching the photocells. One important thing is that whatever system is chosen it should occupy a minimum of space and be as simple as possible. The other important considerations in the design of a reading system are: (1) index width and its

alignment, (2) the viewing angle of the detectors, and (3) cross-talk between zones.

Index width may be regarded as equivalent to brush width in its effect on reading resolution. It is obviously necessary to restrict the effective index width to the width of the smallest division in the code pattern. However, it can not be made small for then the transmitted light flux would be too small to be effective. The ring-to-ring alignment of the index also enters into the problem of reading. In general the code pattern is constructed in such a fashion that it can be properly read only by an index which is exactly perpendicular to the rings. Any deviation from this adds to the equivalent width of the index and also to the ambiguity in reading the segment boundaries.

Cross-talk is the transmission through a zone of the pattern to an adjacent detector. The position and length of the light source must be such that this cannot occur. Each code plate and detector assembly has an acceptable area in which the source, depending on whether it is a point source or a line source, can be located so that there is no cross-talk. This distance from source to code plate must be accurately adjusted.

2. BALDWIN AND DATEX ENCODERS AND THEIR COMPONENTS

a. Baldwin Programmer and Position Encoder

This section is devoted to a description of the Baldwin Model A9SP16 Encoder and its auxiliary equipment (see Illus. 1 and 2). One of the disadvantages of the Baldwin encoder is its dependence on a programmer which supplies a pulse to trigger the flash lamp and a power supply for the transistor amplifiers. The accuracy and reliability of the encoder is no better than its auxiliary equipment and electronic circuitry.

The discussion of each unit follows in their order from left to right in Fig. 4.

The Baldwin Encoder Programmer and Power Supply (Model B/M 907-10)

The programmer section supplies a large positive pulse of voltage which is capable of causing the strobotron transformer to fire the lamp at the prescribed time. This is accomplished by feeding the programmer an external negative trigger pulse of 10 volts magnitude with a 3 micro-seconds duration. This pulse is first amplified into a 14 volt, 3 micro-second pulse. This pulse is then fed into a one shot multivibrator which in turn produces a positive pulse of 60 volts and 82 micro-second width.

This pulse is differentiated, then the positive portion is clipped by use of a diode. The remaining pulse is a negative pulse of 50 volts, which lags the "external trigger input pulse" by 82 micro-seconds. This delayed pulse is then used to cut off a beam power tube which is drawing a quiescent current of 100 ma. The plate cap of this tube is connected to B+ through a transformer located in the encoder. The magnitude of the pulse is about 700 volts. This pulse is stepped up through the transformer to a potential of 4200 volts which is used to trigger the strobo-

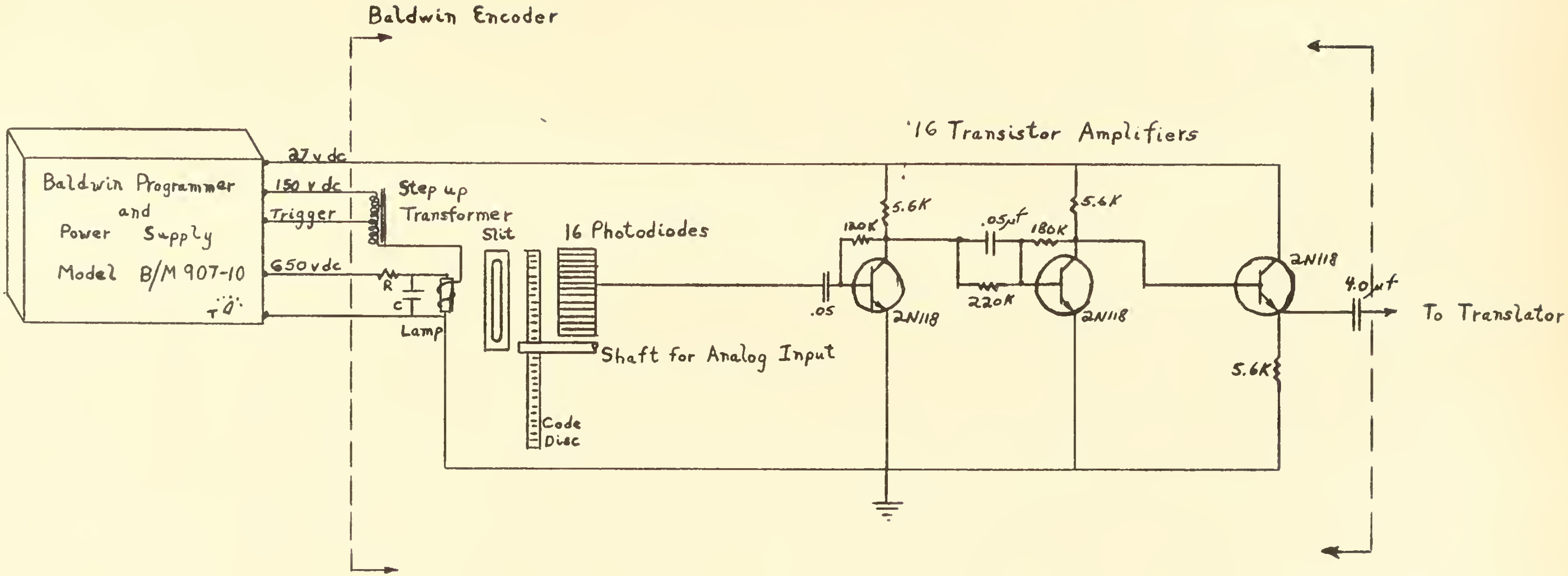


Fig. 4

Circuit Diagram for Baldwin Programmer and Encoder

tron lamp.

Power Source Requirements

Volts	Frequency	Volt Amperes
120	60	60
-10 peak to peak	Repetition Rate[2]	
	$\frac{1}{2}$,5,10,60,100 pulses per sec	

Output Voltages

Voltage	Current	Function
27 D.C.	200 ma	B+ for transistor amplifiers in encoder
150 D.C.	100ma	B+ for tubes in programmer
650 D.C.[3]	10ma[4]	Anode supply for strobotron
3 A.C.	---	To operate Gurley Reader
700 pulse	---	Trigger for exciting strob lamp

Power Supply

The power supply D.C. voltages are obtained by use of transformers and solid state rectifiers. The 150 volt and the 27 volt supplies are both derived from full wave bridge rectifiers coupled to choke input filters. The 27 volt supply has two filter sections, which reduce the 120 cps ripple to approximately 0.1%. The 650 volts supply is derived from a half wave bridge and a large capacitor for filtering. This voltage is variable for the purpose of increasing the intensity of the strobotron lamp when necessary.

[2]The repetition rate potentiometer was changed to a variable position selector in order that any desired sampling rate from 0 - 100 could be obtained.

[3]The 650 volts is adjustable to 800 volts by variac to maintain the proper light output of the strobotron lamp as it ages.

[4]Maximum current required for 100 flashes/sec.

High Intensity Strobotron

Sylvania Strobotron Type S413 is a high intensity strobotron which produces bluish white light pulses at frequencies below 100 flashes per second. It was designed to fill the need for a reasonably priced, compact, slow rate strobotron suitable for true color viewing of relatively low frequency rotary and reciprocatory motion.

The strobotron has a straight tubular configuration. Two electrodes, an anode and cathode, are mounted in the end caps and a trigger electrode is fastened external to the envelope. Ionization is initiated by a potential between the trigger and cathode electrodes. The strobotron consists of a glass or quartz envelope filled with xenon. This gas is most often used because the emission spectrum from xenon is much like that of sunlight. Therefore, it is spectrally compatible with the sensitivity of the photodiodes used in coder applications.

A trigger pulse is formed by discharging a small capacitor through the trigger coil that is wrapped around the lamp. The discharge is initiated by a thyratron tube driven by the control circuit in the Baldwin programmer. Upon initiation of ionization, electrical energy from the storage capacitor in the programmer or the encoder discharges into the gas and creates an intense amount of radiant energy. Once the energy stored in the capacitor is discharged through the gas and recombination is complete, ionization can be initiated again. The energy jump which occurs when the gas ions recombine causes the emission of photons in and around the visible spectrum.

Strobotron Firing Circuit

The following analysis of the firing circuit was necessary in order to determine if any error or inaccuracies could result at the various sampling

rates. The highest sampling rate permissible in the Baldwin encoder is 100 pps. The circuit appears as follows

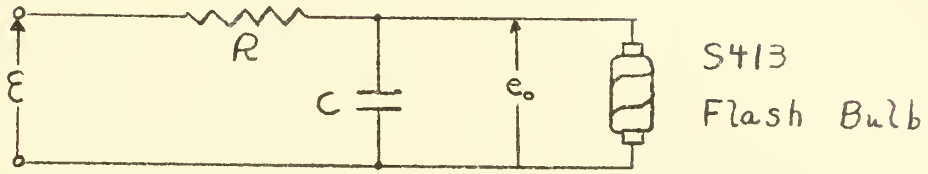


Fig. 5
Strobotron Firing Circuit

where R = 33K

C = 0.1μf

τ = 3.3 μ sec

E = 650 volts

Therefore $e_o = E\,(1 - \epsilon^{-t/\tau})$

At the maximum rate of 100 pps, t = .01 seconds. This is the shortest time available for charging of the capacitor at the different rates used.

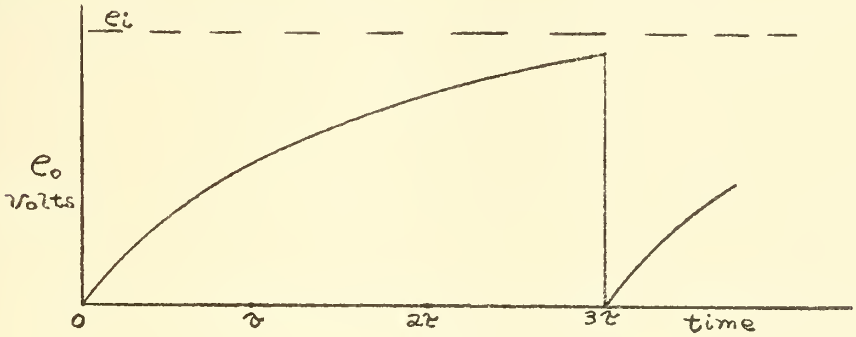


Fig. 6
Capacitor Charging Curve

It can be observed that if 650 volts or slightly less is required to provide sufficient light to activate the photocells there is a possibility of error. Therefore, the variac on the programmer was adjusted to the full value of 800 volts to ensure no error from this source. It was discovered during tests that a voltage above 675 volts was required for no error resulting from insufficient voltage.

Mirror and Aperture

When the strobotron is triggered the light is changed 90° in direction by a mirror, which is on the flash lamp and mirror assembly, into the aperture. The reason for this mirror appeared to be a mounting problem. For future encoders, it is suggested that a new bulb be designed especially for the Baldwin with a direct path through the aperture and housing.

The aperture is 1/8" x 2", the length being horizontal and radial with respect to code disc. A fine quality glass is used in the aperture. This is present to keep the dust and the heat from the lamp from entering the compartment which houses the rotating disc. The tests were accomplished with the electrical compartment housing removed in order that the strobotron could be observed. It was noticed that at the high sampling rates the heat could be significant.

The aperture allows the light to be concentrated on the code disc tracks. To minimize reflection inside the encoder, the lightweight housing of cast aluminum is anodized black.

Code Disc and Slit

Centered on the encoder shaft is the 16 digit code disc, 8 5/16" in diameter, produced photographically on glass by a special divided circle machine. The disc consists of 65,536 (2^{16}) discrete steps, the boundaries of these steps being accurate to better than 5 parts per million in relation to each other. An extra (17th) track is the reference track. This in conjunction with an extra photocell provides a signal for possible use in readout circuitry to compensate for variations in light intensity and in photocell signals with temperature.

Light passing through the code disc is read by a bank of radially

positioned photocells, one for scanning each track on the code disc. Clear sectors of the disc give a "1" output, while opaque sectors give a "0" output. As the shaft is rotated, the disc moves integrally with it, changing the binary word output of the encoder. Counterclockwise rotation, as viewed from the shaft end, produces an increasing output count.

Between the code disc and the photocells is another section of housing with a slit only 5 microns in width which concentrates the light on the proper photocells.

Photocells

For scanning each track on the code disc, P-N junction type photocells, tested and selected for their uniform signal output, size and temperature characteristics are employed, firmly fixed in a removable block. The junction of each photocell is precisely located with respect to the read-out slit. Photocell life is anticipated to be beyond some 50,000 hours of operation.

The voltage output for a binary "1" was found to average 30 millivolts and for a binary "0" about 5 millivolts.

Encoder Transistor Amplifiers

Photocell outputs are coupled to compact, three stage transistor plug-in amplifiers contained within the encoder housing. The amplifiers convert the current pulse from the photocells into suitable output voltages of 6 ± 2 volts.

An analysis of the amplifiers follows in order to check the design and its limitations.

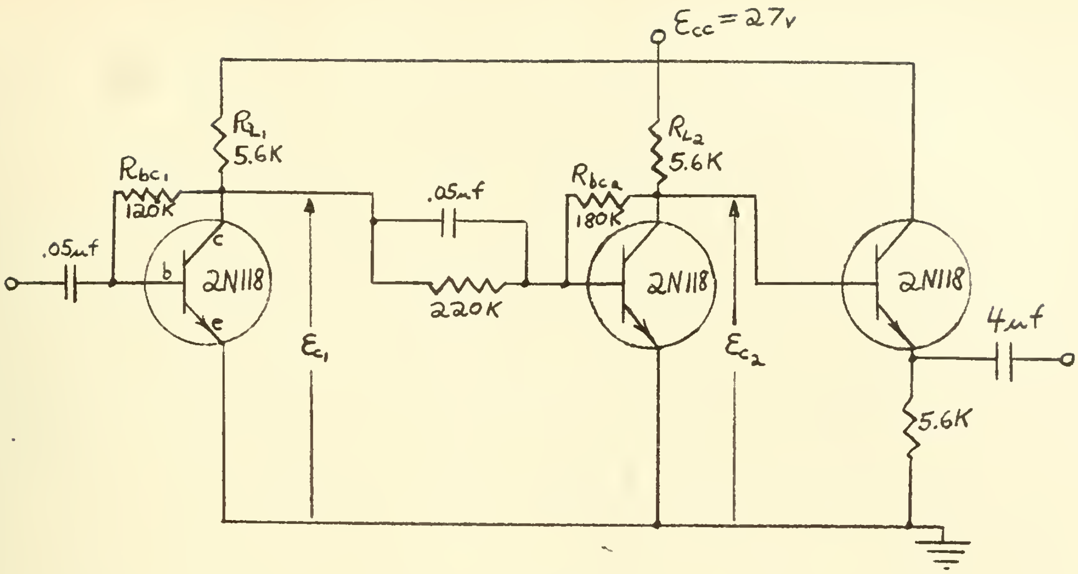


Fig. 7

Encoder Transistor Amplifier Circuit

The encoder amplifier consists of three stages. The first and second stages are self biasing amplifiers employing d-c feedback and the third stage is an emitter follower. The 2N118 transistor is a N-P-N grown junction silicon type designed for high gain at high temperatures. The Beta spread is 18 to 40 where Beta is called the transport factor, i.e. the ratio of holes reaching the collector to the total number of holes injected into the base.

Since proper d-c biasing is used, the transistor amplifier can be expected to operate satisfactorily over a wide range of temperatures and various transistors may be used interchangeably. Ordinarily it is the bias current in which one is interested and which must be adjusted so as to determine the proper operating point. A suitable operating point is made with due consideration of the available supply voltage and current and of the magnitude of the signal to be amplified.

Referring to the characteristic curves for the 2N118 transistor and

the circuit diagram the approximate magnitudes of currents and voltages can be calculated.

$$E_{c_1} = \frac{E_{cc}}{1 + \frac{R_{L_1}}{R_{bc_1}}(1 + \beta)} \qquad (1)$$

Solving, $9.4 < E_{c_1} < 14.4$ volts

An Average $E_{c_1} = 12$ volts.

The dc bias point is located by the following equation

$$I_{CB_{o_1}} = \frac{E_{c_1}}{R_{bc_1}} \qquad (2)$$

Thus $78.3 < I_{CB_{o_1}} < 120$ microamps

An Average $I_{CB_{o_1}} = 100$ microamps.

It was found that the temperature was not high (about $30^{\circ}C$) in the plug-in amplifiers, therefore the Beta was approximately 22.

The collector current

$$\Delta I_c \cong \beta I_b \qquad (3)$$

A typical input from the photocell is approximately 3.5 microamps, for which

$$\Delta I_c \cong 77 \mu a$$

This current is divided between the path through R_{L_1} and the line to the next transistor.

$$\Delta I_{B_2} = \frac{5.6K}{6.6K} \times 77 \mu a = 65.5 \mu a$$

Therefore, the change in voltage when the pulse is applied is

$$\Delta E_1 = I_{c_1} R_{L_1} = 0.43 \text{ volts}$$

The second stage is essentially the same as the first stage except that

the feedback resistance R_{bc_2} has been increased so that the operating point is lower on the load line. Thus, the amplified pulse from the first stage has sufficient travel on the load line before intersecting the nonlinear point at the knee of the characteristic curves.

Using equation (1)

$$12.45 < E_{c_2} < 17 \text{ volts}$$

$$\text{Average } E_{c_2} = 14.75 \text{ volts}$$

From equation (2) the dc bias point is

$$69.2 < I_{CBo_2} < 94.5 \mu a$$

$$\text{Average } I_{CBo_2} = 81.85 \mu a$$

From equation (3) with $I_{b_2} = 65.5 \mu a$

$$\Delta I_{c_2} = 1440 \mu a$$

The change in voltage when the pulse is applied is

$$\Delta E_2 = I_{c_2} R_{L_2} = 8.1 \text{ volts}$$

Since the emitter follower presents such a high impedance essentially all the current flows through the emitter resistance and the change in voltage at the output is the same as E_{c_2}. The 30 millivolt input has been amplified to approximately 8 volts by the two amplifier stages and the last stage, and the emitter follower presents a low impedance to the translator.

It appears, from a design standpoint, the plug-in transistor amplifier designed by the Baldwin Piano Company is satisfactory for the task it was assigned. Some typical measured values were taken so that the analysis

could be verified.

First Stage

E_{b_1}	0.66 volt
I_{cb_1}	93 μa
I_{c_1}	24 ma
E_{c_1}	11.8 volts
I_{e_1}	2.4 ma

Second Stage

I_{cb_2}	53 μa
I_{c_2}	25.4 ma
E_{c_2}	10.3 volts
I_{cab_3}	108 μa

Third Stage

I_{c_3}	1.47 ma
I_{e_3}	1.6 ma

Baldwin Encoder Output Pulse

During the experimental tests, photographs were taken of the most significant bit output at its lowest voltage. This particular point is the 180° position where all the 2ⁿ bits have a cyclic code output of a binary "1".

Cyclic code: 1,111,111,111,111,111

Normal Binary: 1,000,000,000,000,000

The output pulses (see Illus. 6) have a rise time of less than 1 microsecond and are approximately 2 and 10 microseconds wide at 90 and 50% amplitude respectively. Maximum accuracy in associated readout equipment

occurs when the operating threshold of the readout circuit is established at 50% of the pulse amplitude which should correspond to the center of a transition from a dark to a light area of the code disc.

Normally, the output will be at 50% of the peak amplitude pulse when the slit is exactly half covered by a dark area of track. Maximum accuracy thus is obtained by reading out all transitions at 50% because at that voltage the sharp boundary is centered over the slit.

The reliability factor can be improved by establishing a threshold voltage which is less than 50% of the pulse amplitude, but this results in a loss of accuracy. This loss of accuracy is occasioned because the transitions between light and dark areas are not instantaneous when practical and useful encoder package sizes are considered. It is noted that the finite change in angle necessary to complete a transition is a result of the use of a practical reading slit width and other optical considerations.

Signal variations could be a result of degradation of the strobotron, amplifiers, photodiodes or supply voltages and, although not common, must be considered when maximum reliability is required.

One of the more simple and reliable methods of establishing the readout threshold embodies the use of a silicon diode in series with the encoder output. This diode is biased to the desired level, and by an adjustable bias it is possible to establish the optimum value for each output signal. The signal amplitude may be increased by use of a pulse transformer when increased signal is desired.

b. Giannini Mechanical Shaft Position Encoder

One way to improve the resolution of an encoder is to increase the number of rings on the disc thereby permitting the use of smaller and

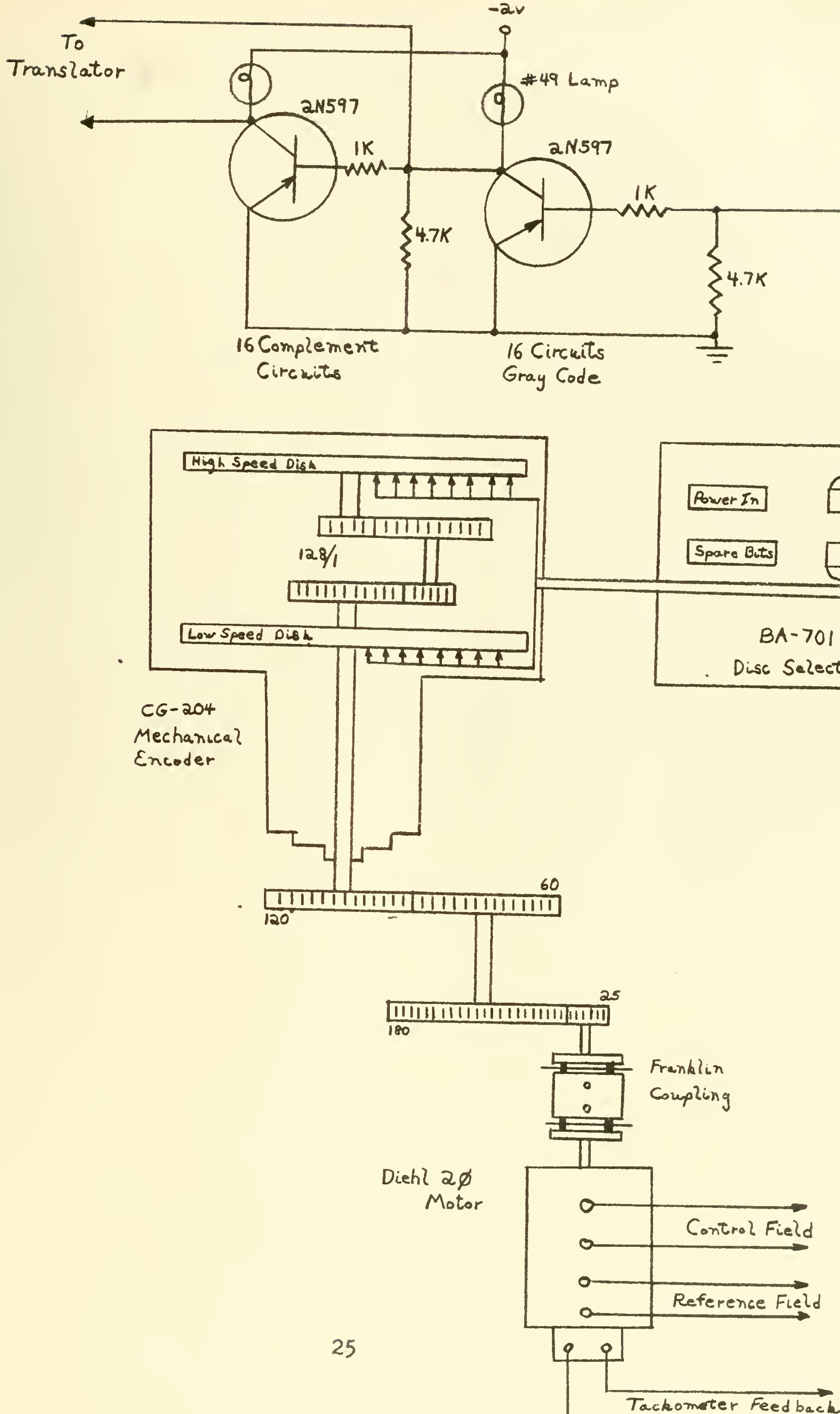
smaller segments. However space considerations or other factors may prevent the use of discs of large diameter. Therefore a second method has been devised, that of using multiple code discs, called multispeed or multiturn encoders. In this system each succeeding disc is geared to turn at a slower speed than the one preceeding it so that the system resolution is still determined by the total number of rings. The slow speed or higher order digit disc is used to count the number of revolutions of the high speed (low order digit) disc.

The system employed for this investigation used a dual-disc Gray code encoder in conjunction with a BA-701 Encoder Disc Selector, both built by the Datex Division of the G. M. Giannini & Co., Inc. The combination of the two discs gave a total bit count of 2^{16} (65,536), the same as the Baldwin model used. The arrangement of this system was as shown in Fig. 8 with the actual physical set-up as shown in Illus. 3. The encoder disc providing the nine least significant digits (high speed disc) is attached directly to the input shaft and therefore provides an accuracy of the order of the disc accuracy itself ($\pm$ 1 quanta). The low speed disc (having the seven most significant digits) is attached to this through a step-down gear ratio and essentially counts the number of revolutions of the high speed unit. The overall accuracy of the system is determined by the accuracy of the encoder attached to the input shaft.

The low speed disc is equipped with two sets of brushes and so phased that one set is leading and the other set lagging the transition point (conducting to non-conducting segment) of its least significant ring by a $\frac{1}{4}$ count. The selection of either leading or lagging brushes is deter-

Fig. 8

Datex Encoder System Connections



mined by the disc selector and is based on the reading obtained from the most significant ring of the high speed disc. Thus, since transition is determined solely by the reading of the high speed disc, simultaneous, exact contact changes between the two discs are not required. Also this permits using a small amount of overlap on the lead and lag segments of the low speed disc thus overcoming any errors due to backlash and gear wear between the two discs.

The BA-701 Encoder Disc Selector (pictured in Illus. 4) will actually receive data from two separate encoders, but was used here with only the one CG-204 model. The Selector takes the encoder output and, through transistor circuitry, performs the lead-lag selections required. It also includes circuitry to eliminate or minimize the effect of brush noise and brush bounce, and thus extend the useful life of the encoder assembly. The output of the BA-701 was fed through an indicator lamp and resistor-transistor circuitry in order to reduce its 10 volts to the maximum allowable two volts of the translator. As in the Baldwin system, the translator was necessary in order to change the Gray code output to binary.

3. AUXILIARY EQUIPMENT

Digital Comparator

The comparator used in this system was the Norden-Ketay Digital Comparator. This model is completely modernized using semiconductors mounted on etched cards of the plug-in type.

The purpose of the comparator is to compare two binary numbers and supply an A.C. output signal. This output signal is a carrier-suppressed amplitude modulated signal with amplitude and phase being proportional to the difference between the input binary numbers. 115 volts A.C. supply must be provided the comparator as a source for this output signal.

A block diagram of the comparator is shown in Fig. 9. The current generators each consists of six transistors with collector load resistors so weighted that each more significant binary digit produces twice as much current as the preceding one.

Logic circuit diagrams are shown in Fig. 10. The purpose of these circuits is to close the right combinations of transistors in the + and - current generators to produce an output A.C. voltage that is proportional to the difference between their input numbers. This logic circuitry is called "current" logic since it sets up a single current path through a number of possible paths. The transistors are thus, really, just open or closed switches. The logical levels used are:

$$-11 \text{ volts} = 1 = \text{true}$$

$$0 \text{ volts} = 0 = \text{false}$$

A logical proposition (a binary digit or bit) may be indicated as A_4. The inverse of this is then written $\overline{A_4}$. Thus if $A_4 = 1 = -11$ volts, then, simultaneously, $\overline{A_4} = 0 = 0$ volts.

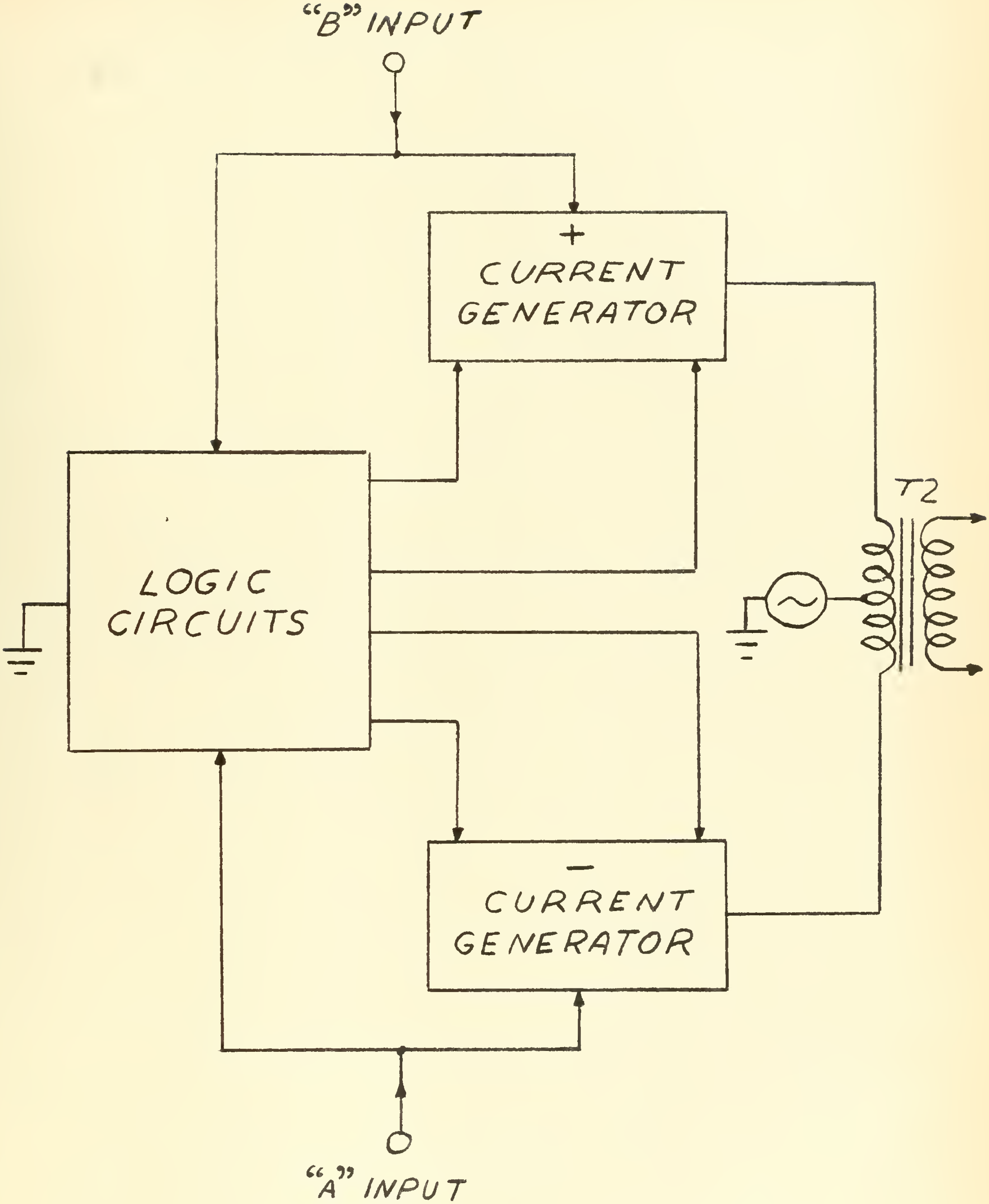


FIG. 9
BLOCK DIAGRAM COMPARATOR

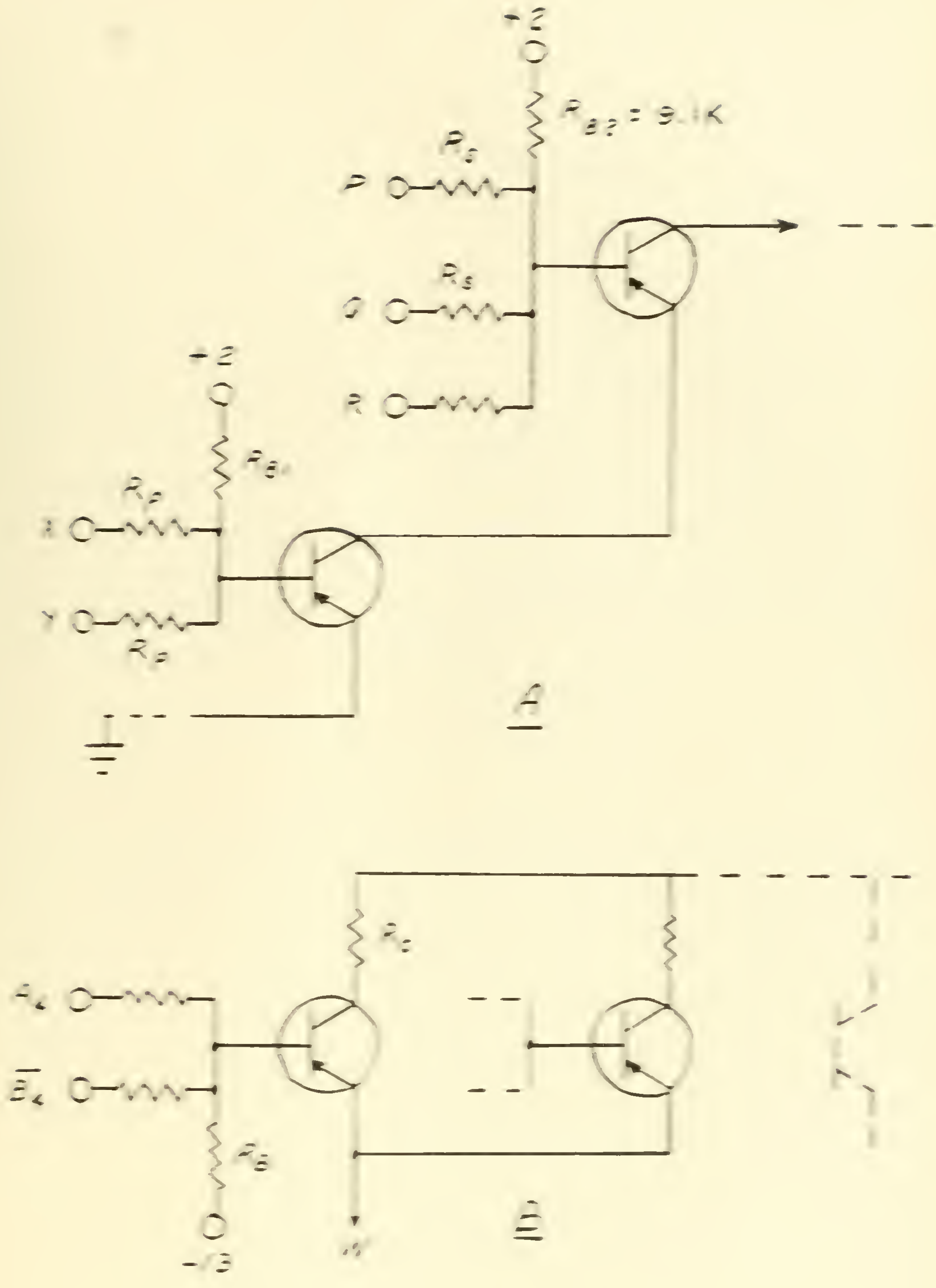


FIG. 10
TYPICAL LOGIC CIRCUITS

Two types of enabling circuits are found in the logic portion: a logical "product" (called an "and" circuit) and a logical "sum" (called an "or" circuit). To identify one from the other, check the value of the transistor base bias resistor. If this is 9.1K the transistor switch is enabled by the logical sum of the terms appearing on its base; if less than 9.1K a logical product is being formed. In Fig. 10A a typical circuit is found expressing the logical Boolean current path equation XY (P+Q+R).

Fig. 10B is a typical switching arrangement found in the current generators. To enable this switch it is necessary that both A_4 and $\overline{B_4}$ be at 0 volts, or, in other words, $\overline{A_4}$ and B_4 are -11 volts.

The basic idea behind the comparator operation in performing its function of converting two input digital signals to one output A.C. voltage is best understood by use of an illustration. To do this let us assume that the two input numbers are

	8	4	2	1
a =	1	0	0	1
b =	1	1	0	0

Here the rows indicate separate numbers while the columns of digits represent binary bit positions of eight, four, two and one, respectively reading left to right. Current flow through channels controlled by each column is weighted proportionately the same way, i.e., 8, 4, 2 and 1. Now it is noted that in the 8 column for these numbers both a and b have the value 1. Thus, in the circuitry arrangement in this comparator, there is no current flow. In the 4 column a is 0 and b is 1, therefore b is greater than a and the current value is proportional to, say, -4. In the 2 column both are 0 so again there is no current flow. In the 1 column

a is greater than b and the current value is, therefore, +1. Summing the currents of the four channels gives -4 plus a + 1, or, a final value of -3. Note that algebraic summing of the two binary numbers does, in fact, give -3:

	binary		decimal
a =	+ 1001	=	+ 9
b =	- 1100	=	- 12
	sum		- 3

This comparator is so designed that the logic circuitry described above only functions for values of 2^5 or less, i.e., the last five channels. This means, basically, a saving in the amount of logic circuits needed. Operation beyond the point of saturation (2^5) is channeled in accordance with the difference (called "d") between the two numbers, working from the most significant channel down. If the two digits are the same d = 0 and a current path called the center channel is energized. Travel is down this channel until a difference is noted, either 1 or 2 as to whether or not a is greater than b or b greater than a, whence flow is diverted through another path. In this manner voltages are generated to rapidly bring system correspondence down to the 2^5 level.

The voltage level for each bit or quantum up through the first 32 is shown in Fig. 11.

Translator

The translator is shown in Illus. 5. Its function is to translate the Gray binary output of the encoders into a natural binary input to the comparator. The original unit, designed for the Baldwin encoder, whose output is sampled pulses, makes use of two basic circuits: flip-flops

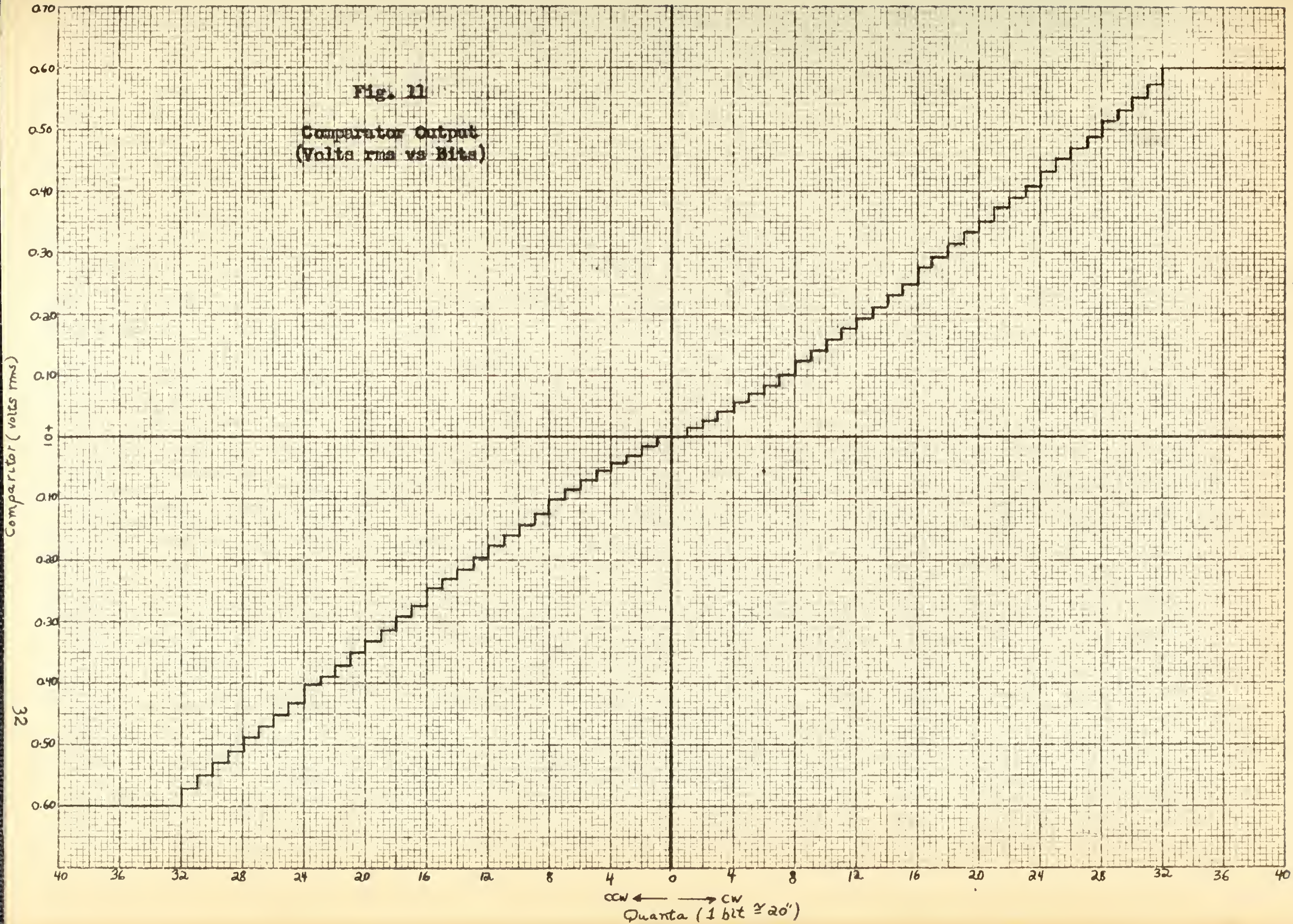


Fig. 11
Comparator Output
(Volts rms vs Bits)
Comparator (volts rms)
0.70
0.60
0.50
0.40
0.30
0.20
0.10
10+
0.10
0.20
0.30
0.40
0.50
0.60
40 36 32 28 24 20 16 12 8 4 0 4 8 12 16 20 24 28 32 36 40
CCW ← → CW
Quanta (1 bit ≅ 20″)

and gates. The flip-flop transistor "memory" circuit holds the pulse from one encoder track for comparison with the translated signal from the previous track in the gate circuit. With the decision to use the translator for the Datex encoder as well it was necessary to modify this arrangement. Since the output of the sixteen Datex tracks is continuous it was necessary to by-pass the flip-flops and take their output directly to the gate circuits. In order to permit ease of shifting between the two encoders the switching arrangement of sixteen relays (see Illus. 3) was installed between the encoders and the translator.

The translator consists of sixteen boards, one for each binary number or track of the encoder. Fifteen of these boards, from the second most significant digit to and including the least significant digit, are identical in construction (see Fig. 12). In converting the input signal from Gray to natural binary form each of these boards is dependent upon the output of the previous board as well as its own encoder track input. This is not true of the board representing the most significant digit. Since it has no previous board its output is dependent solely upon the input from the encoder's most significant track and is therefore read directly, with no conversion necessary, (see Fig. 13).

The principle underlying the operation of the translator boards is best understood by considering the following examples. The first example illustrates how a binary set of digits can be easily converted into a Gray (cyclic) set by taking the most significant digit (one to the left) adding it to the adjacent digit to the right, and placing the result below the latter as its Gray equivalent. The identical procedure is followed for the second and third digits, and so to the end.

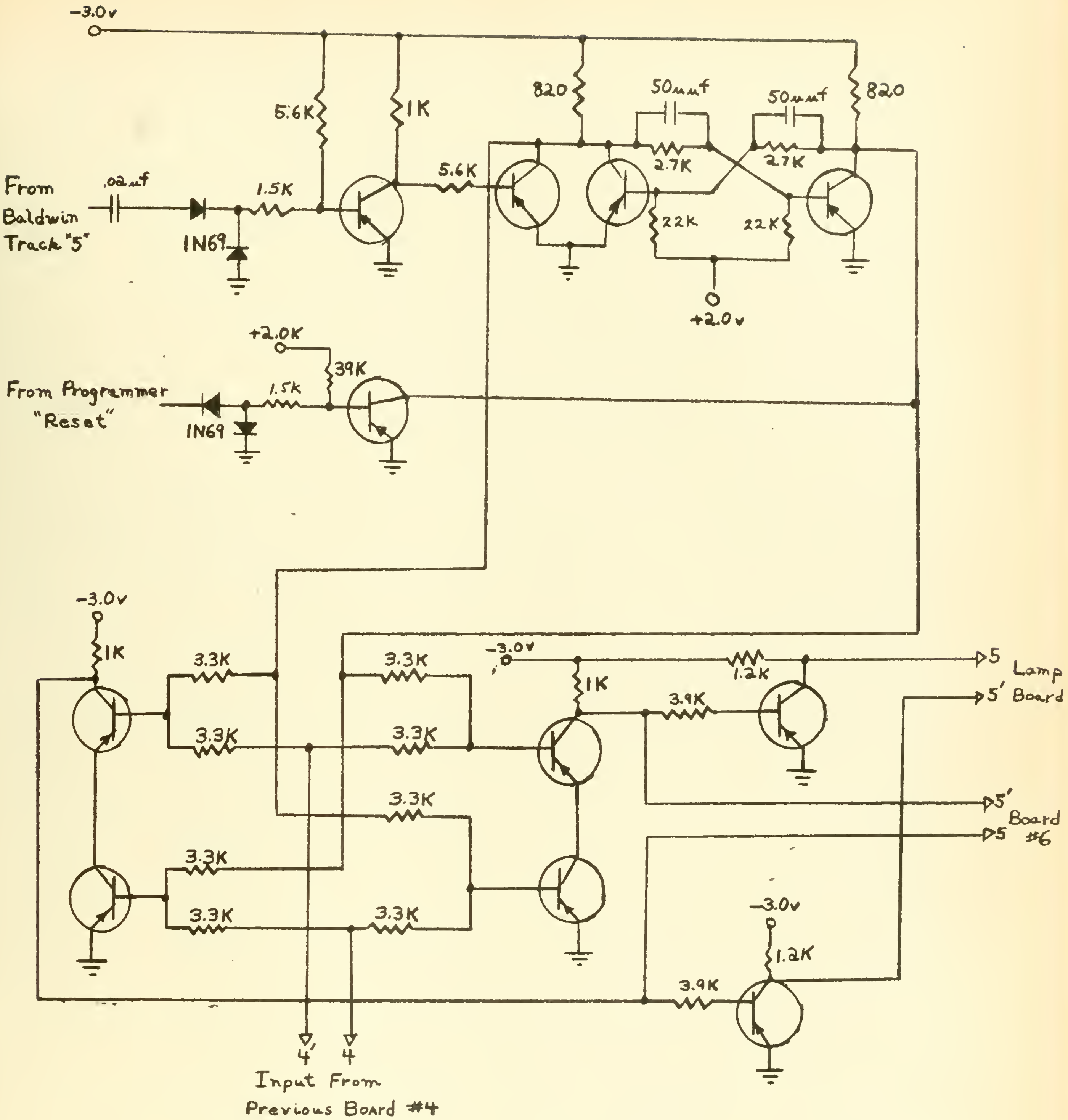
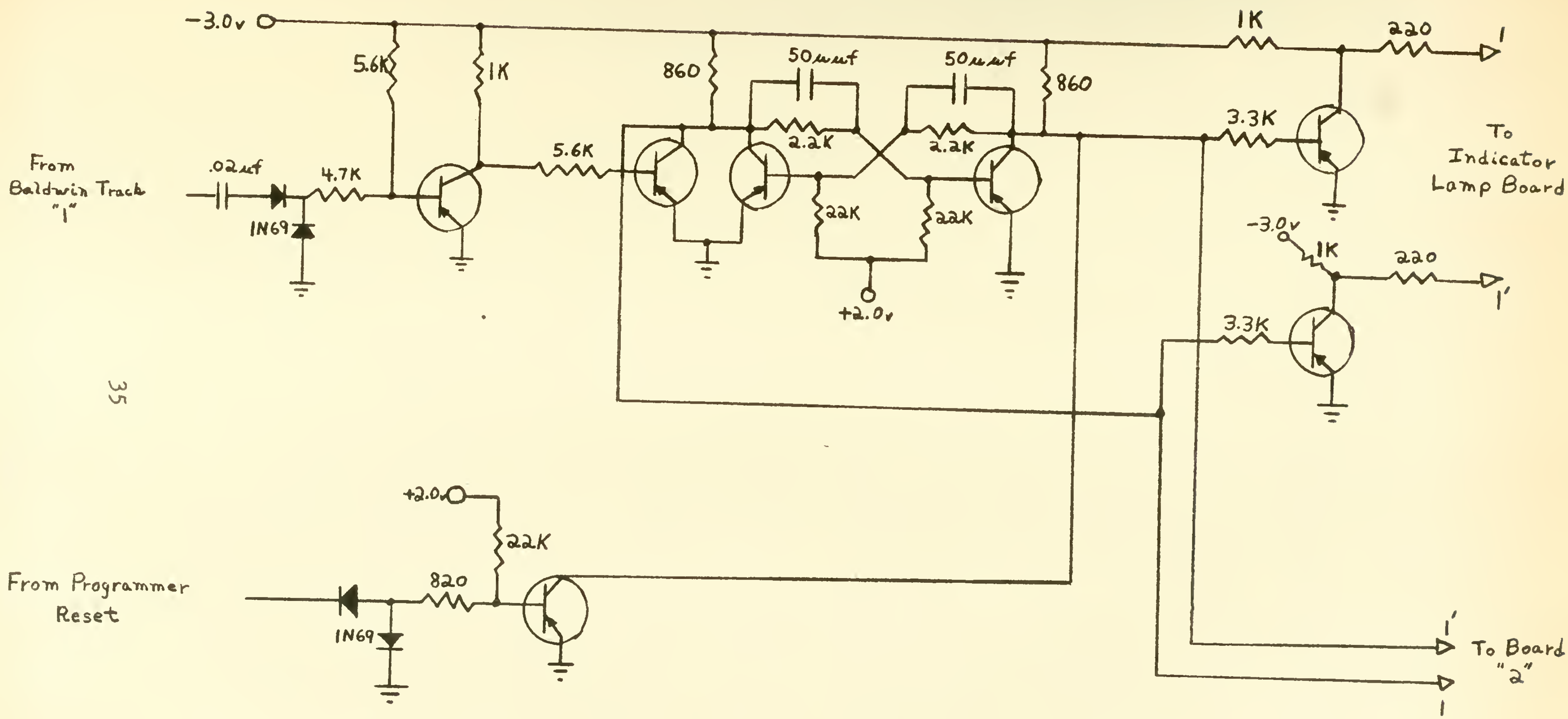


Fig. 12

Storage Register and Translator Circuits
of all Boards but Most Significant Bit



All Transistors are 2N344

Fig. 13

Storage Register and Translator Circuits
for Most Significant Bit Board

$$\begin{array}{lccccccccccccc} \text{Binary number:} & 1 & \rightarrow & 0 & \rightarrow & 1 & \rightarrow & 0 & \rightarrow & 1 & \rightarrow & 1 & \rightarrow & 1 \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{Gray (cyclic) equivalent:} & 1 & & 1 & & 1 & & 1 & & 1 & & 0 & & 0 \end{array}$$

It is to be noted that 1 + 0 = 1 and 1 + 1 = 0 in this procedure. Also observe that the most significant digit was carried down as given in the binary number.

For converting this number back to binary from the Gray the procedure followed is illustrated by the arrows in the following example:

$$\begin{array}{lccccccccccccc} \text{Gray number:} & 1 & & 1 & & 1 & & 1 & & 1 & & 0 & & 0 \\ & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow & \nearrow & \downarrow \\ \text{Binary equivalent:} & 1 & & 0 & & 1 & & 0 & & 1 & & 1 & & 1 \end{array}$$

It is readily noticeable that in both examples the most significant digit determines itself, however all others are determined by combining two values. In the latter case, Gray to natural binary translation, these two values are the output of the previous translation and the incoming Gray bit.

To accomplish this translation the boards, whose circuit diagrams are illustrated in Fig. 12 and Fig. 13, were designed. The first of these is for the most significant digit and consists primarily of a transistor flip-flop circuit. This circuit is designed to give an output signal of the proper magnitude for comparison purposes in the comparator and for use in the next translator board. The action of the flip-flop is such as to be essentially independent of the amplitude of the input triggering pulse once triggering action has taken place. The second figure illustrates the circuitry used for all other digits in the translator. It consists basically of a flip-flop circuit and a gate or logic circuit. The flip-flop is employed here as in the first board but its output, with its "memory" characteristic, is sent through the gating circuit. Here it is combined

with the incoming signal from the previous board to determine the proper binary signal output.

The output of each board is sent to an indicator lamp on the face of the translator as well as to the next board. When a lamp is lighted it is counted as a binary 1, when out, it is a binary 0. The condition of the sixteen lights on the face of the translator was used as an indication of system final error. Any binary count remaining after steady-state was reached indicated disagreement between the encoder position and the system input signal from switch bank "A" or "B" (shown on Illus. 2), in other words, an error.

Servo Motor

An A.C. low inertia servo motor with D.C. tachometer was purchased from Diehl Manufacturing Company for the system.

The low inertia servo motor is a device used to obtain rapid application of mechanical torque proportional to an electrical input. It is a two phase induction motor constructed with a high resistance rotor having a small diameter-to-length ratio so as to keep the moment of inertia of the rotating member small.

The motor will respond to changes in the amplitude of the applied inputs and to changes in the electrical phase angle between the inputs. The application is with continuous excitation on one winding (called the reference winding) and excitation varying in amplitude, but constant in phase, on the second winding (called the control winding). The motor is reversed by reversing the polarity of the excitation to either winding.

When a two phase motor with a high resistance rotor is used with constant amplitude excitation of the reference winding and variable ampli-

tude excitation of the control winding in time quadrature with the reference voltage, the speed-torque curves are essentially straight.

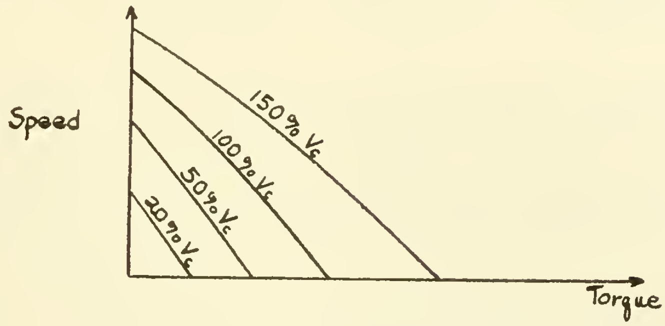


Fig. 14

Servo Motor Performance Curves

Decreasing the control phase voltage (V_c) produces a family of speed-torque curves. V_c = 115 volts for this particular servo motor.

This type of servo motor is designed to give a linear variation of locked torque (that torque exerted by a rotor which is physically restrained from turning) with control field voltage. This linear relationship is valid until the effects of increasing iron saturation are introduced by the higher excitation. From the two curves (the locked torque and speed versus control field voltage plotted from data obtained from test runs performed on the motor) the engineer is able to calculate the transfer function of the motor. Using these curves the transfer function of the motor was found to be

$$\frac{K_v}{S(\tau_m S+1)} = \frac{3.08}{S(.0306S+1)}$$

Calculation of the motor transfer function can be made directly from the performance data sheet that arrives with the motor. Referring to this data the time constant τ_m and K_v can be computed in the following manner.

$$\tau_m = J \left|\frac{\Delta S}{\Delta T}\right| = \frac{\text{Speed}}{\text{Acceleration}}$$

$$J = 0.180 \text{ oz-in}^2 \text{ or } 2.42 \times 10^{-6} \text{ slug ft}^2 (\text{lb ft sec}^2)$$

Using the RPM versus torque curve given on the performance data sheet, the slope is found to be

$$\frac{3450 \text{ rpm}}{5.5 \text{ oz in}} \quad \text{or} \quad 1.26 \times 10^4 \text{ rad/lb ft sec}$$

$$\tau_m = (2.42 \times 10^{-6} \text{ lb ft sec}^2)(1.26 \times 10^4 \text{ rad/lb ft sec}) = .0305 \text{ sec}$$

$$K_v = 1/K_e \quad \text{where } K_e = \text{voltage speed coefficient}$$

$$K_e = \frac{115 \text{ volts}}{361 \text{ rad/sec}} = 0.319 \text{ volt/rad/sec}$$

$$K_v = 3.14 \text{ rad/volt sec}$$

Therefore, the calculated transfer function is

$$\frac{K_v}{S(\tau_m S+1)} = \frac{3.14}{S(.0305 S+1)}$$

Servo Amplifier

Accuracy specifications generally determine what type of amplifier is required in a servo control system. In this case, it was necessary to use a high gain amplifier in as much as the output of the comparator was only 20 millivolts for a one bit error and the system was required to be accurate to $\pm$ 1 bit. Consequently, Philco selected an amplifier capable of a gain in the order of a million to one.

Referring to the circuit diagram of the servo amplifier (Fig. 15), there are two inputs to the amplifier; the 60 cycle ac, amplitude modulated voltage from the comparator and the dc tachometer feedback voltage. The latter is converted to an ac signal by a mechanical dc chopper at a 60 cycle rate synchronized with the reference ac voltage of the motor. This crude ac signal is smoothed and slightly amplified in the next two stages. A potentiometer was installed following these stages in order to control the degree of tachometer feedback voltage.

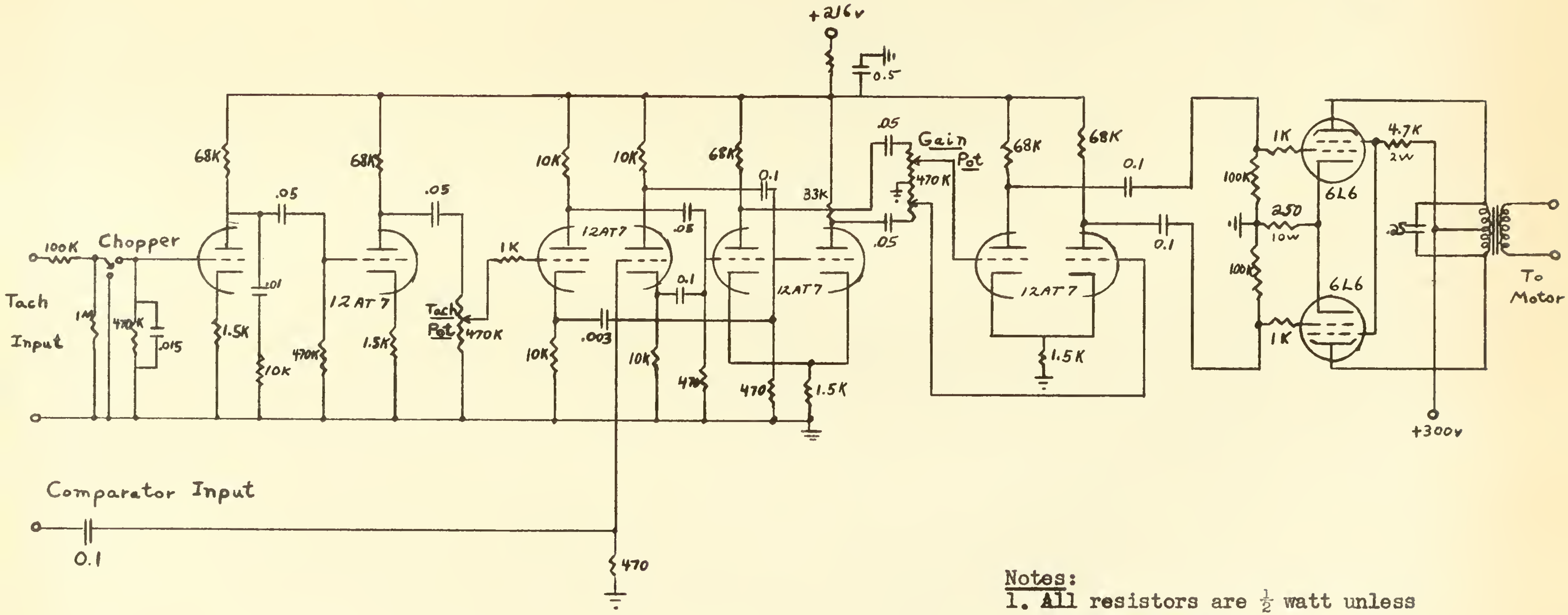
Comparator input enters midway in the circuit, in as much as the tachometer becomes an ac voltage in this area. Summing of these two signals is performed at the gain potentiometer which controls the amplifier output voltage. The remaining stages simply amplify this selected ac voltage. The 0.25 mfd capacitor across the terminals of the transformer shifts the phase of the ac voltage by 90 degrees to provide the proper control field voltage for the 2 phase Diehl motor.

The output transformer and capacitor form a basic RLC circuit which could possibly cause an AC transient in the amplitude of the AC output voltage. To test the amplifier for this transient, a 0.6 rms AC voltage was applied instantaneously by closing a toggle switch. The input and output of the amplifier were recorded on a brush recorder at 250 mm per second. At this recorder speed, no AC transient could be noticed.

A second test was performed on the amplifier to verify its designed high gain characteristics. The tachometer feedback voltage was disconnected and a potentiometer was placed across the comparator output in order that the input to the amplifier could be varied. Various amplifier output voltages were recorded for the associated inputs and the result

Fig. 15

High Gain Servo Amplifier Circuit Diagram



Notes:
1. All resistors are $\frac{1}{2}$ watt unless otherwise noted.
2. All capacitors are in μf.
3. Chassis ground at input only.

was plotted in Fig. 16. Obviously, this is a typical saturation curve. Saturation is present when the output fails to continue to increase proportionately to the input and the effective output to input ratio is reduced.

A system which saturates, usually has as its most common effect, a persistent oscillation. Thus, the output oscillates or hunts around the desired value.

Sometimes this is advantageous, for it permits a simple control system. A limit is placed on the error signal, and the output oscillates about the desired input value. A system of this type possesses both a very fast response (due to the high gain) and simplicity. It has the disadvantages of a steady-state oscillatory output and the accompanying equipment wear and waste of power. However, here the oscillation was overcome by the addition of tachometer feedback.

The overall gain was determined to be approximately (10^4-10^5).

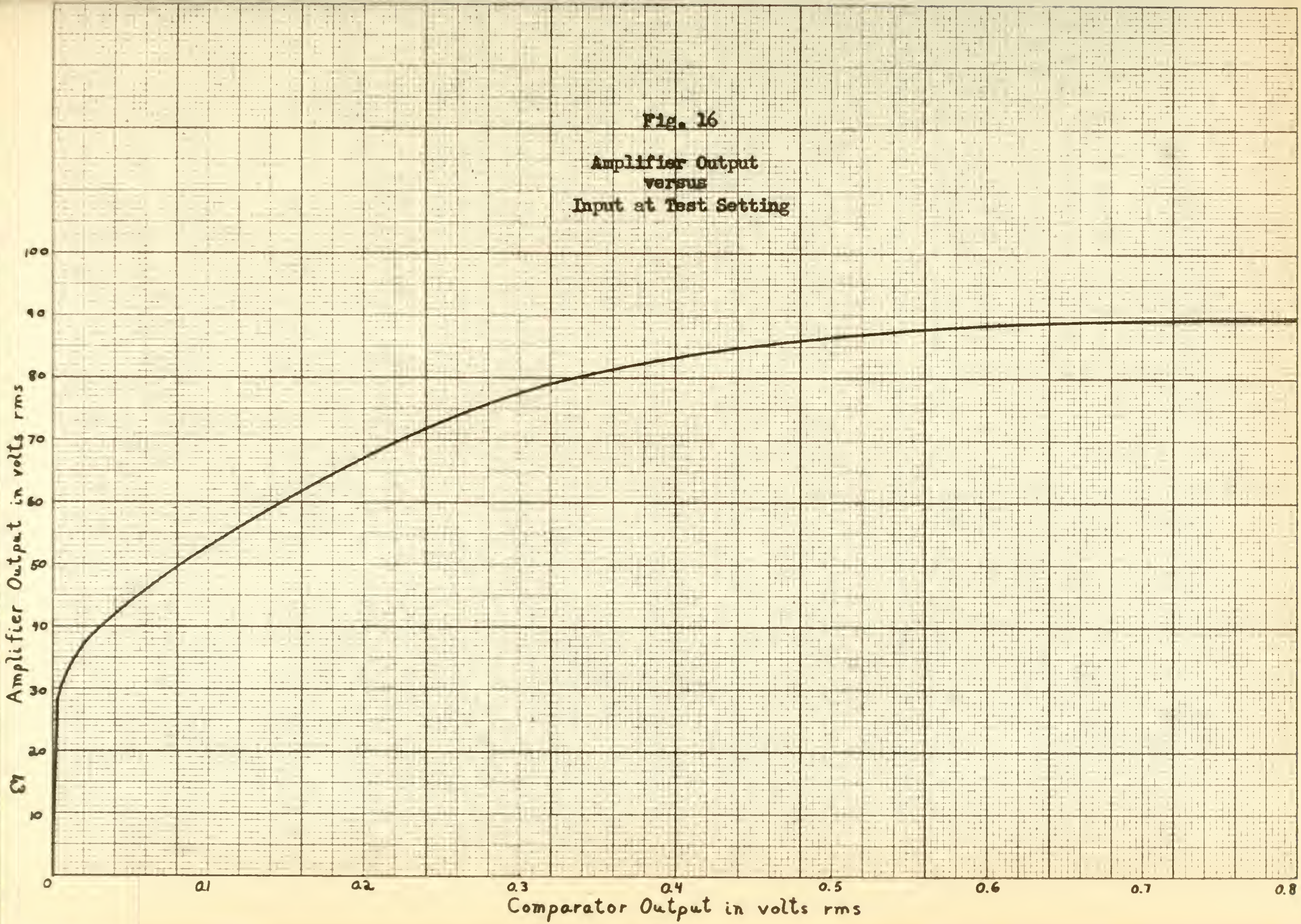


Fig. 16

Amplifier Output
versus
Input at Test Setting

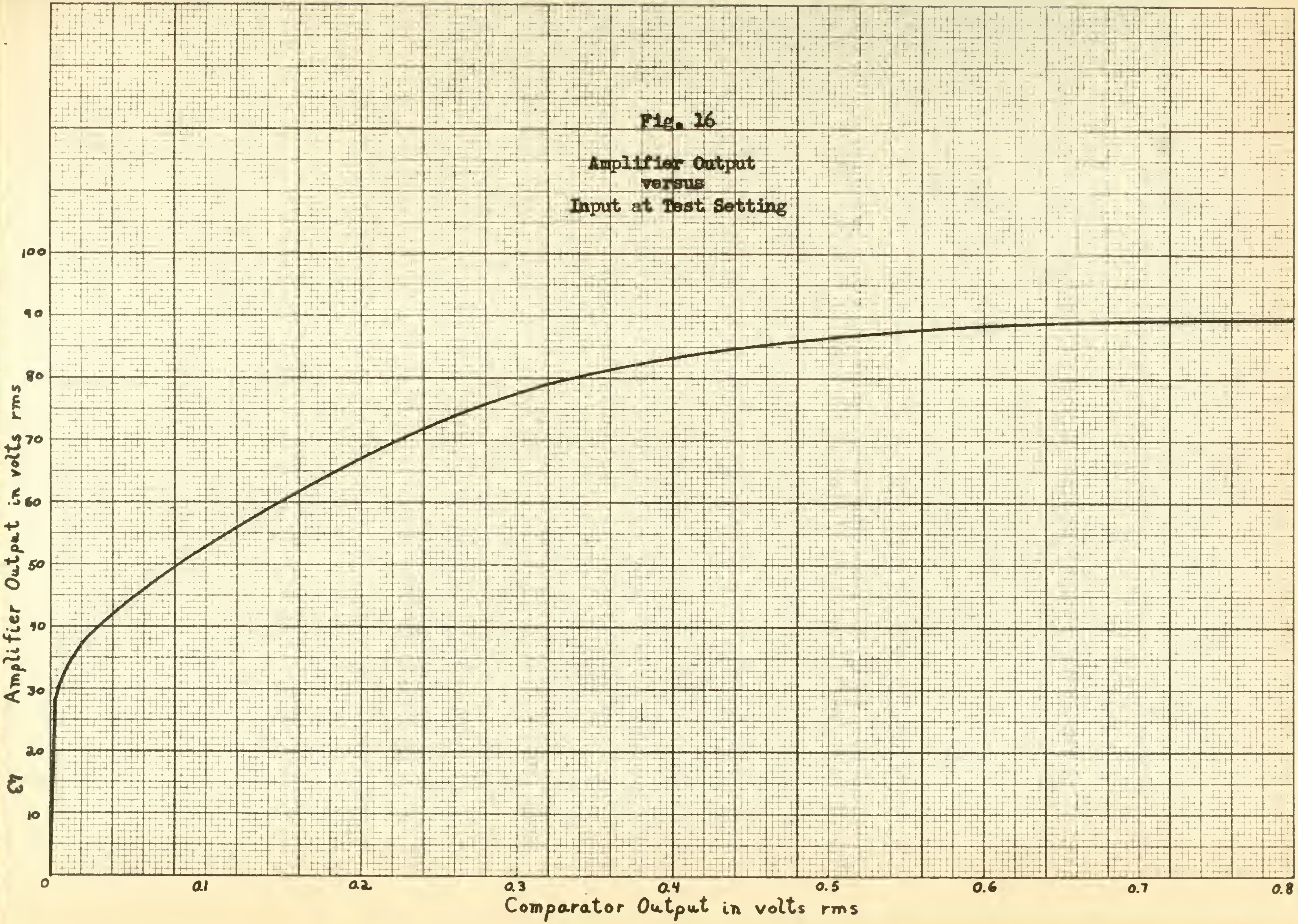


Fig. 16

Amplifier Output
versus
Input at Test Setting

4. ANALYSIS

a. Mathematical Analysis of Baldwin Encoder and System

There is a very fundamental problem which applies to our consideration of a physical system, and not exclusively to those which employ analog-to-digital techniques. The problem is that one does not know in detail how closely a physical system acts like a mathematical model which one constructs to describe it. In particular this is true of the problem of approximating a physical system by models which assume linear behavior. Where such an approximation is quite close, the mathematical techniques will furnish results which are in close agreement with actual behavior. In this control system there is no guarantee that the operation of the system will not enter into non-linear regions. Every possibility was examined to determine these regions in order that the behavior might readily be explained.

The comparator and the high gain servo amplifier are both non-linear components. The comparator saturates whenever the error is greater than 2^5 bits (10.54 minutes) and the resulting comparator output is a continuous signal of constant amplitude until the error decreases below this value. The amplifier has a large gain of approximately 10^4 to 10^6 and thus saturates immediately at one or two bits. This is necessary in order to obtain the $\pm$ 1 bit accuracy required for Philco's large tracking radar. With gains in the normal range of 100-500, the voltage which is applied to the control field of the servo motor by the amplifier for an input of one or two bits would not be sufficient to cause a motor response.

Saturation combined with tachometer feedback is used to advantage in improving system synchronizing performance. The most common effect of

saturation is that it causes a persistent oscillation in the system with a normal signal input. Thus, the output oscillates or hunts around the desired value. The tachometer feedback dampens these oscillations and it can be noted that it is possible to have underdamping and overdamping as a result.

The gear train had considerable backlash, this combined with the saturation effects could result in continuous oscillations. It was necessary to have considerable tachometer feedback with the ability to control it in order to eliminate or dampen the oscillations. These oscillations occur only in the 2^5 bits region; therefore, the analysis will concern itself here, since the comparator saturates at this point.

The transfer function of the Encoder - Translator can be assumed to be unity since they convert a signal from one form to another with a negligible delay (approximately $25\,\mu$ sec). Since the Baldwin Encoder acts like a sampler this results in the system being a sampled data one with additional non-linearities.

The comparator has two functions: 1) to take the difference of the input and output and to amplify this error in the form of a 60 cycle a-c voltage (K_c = 187.5 volt/rad), 2) to act as a clamper or zero order hold circuit in the region below 2^5 bits.

With the preceding introduction, the block diagram can be expressed as:

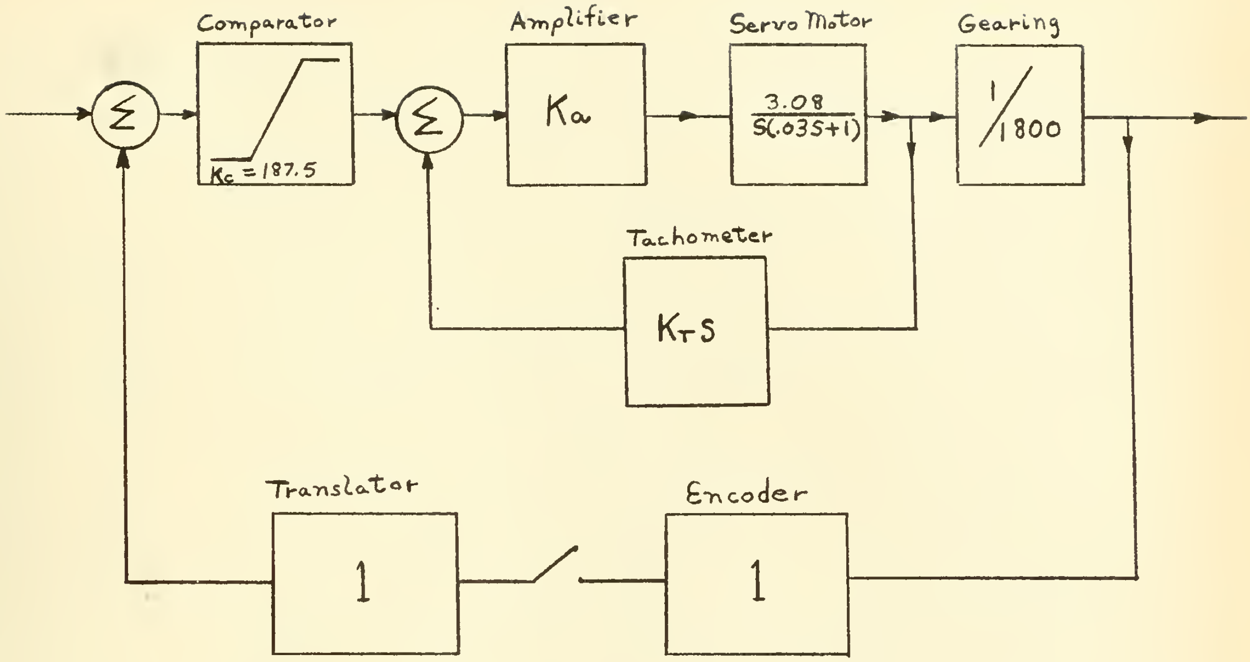


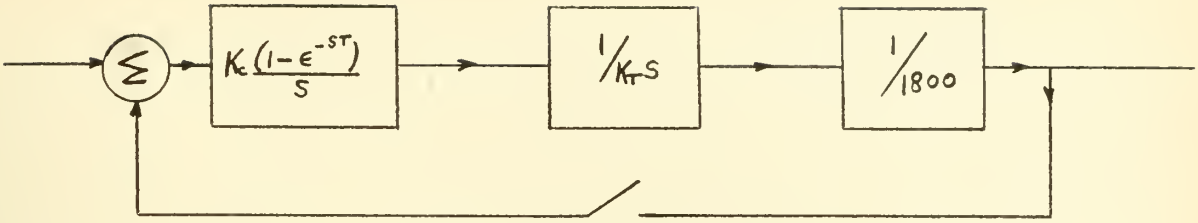
Fig. 17

Baldwin Encoder Servo System

Since the amplifier saturates immediately with its high gain, the inner loop can be reduced in the following manner. Combining the inner loop block to obtain an overall inner loop transfer function:

$$\frac{\frac{3.08\,K_a}{S(.0306S+1)}}{1+\frac{3.08\,K_a}{S(.0306S+1)}K_T S} = \frac{3.08\,K_a}{S(.0306S+1)+3.08\,K_a K_T S} = \frac{\cancel{3.08\,K_a}}{\cancel{3.08\,K_a}\,S\left[\frac{.0306S+1}{3.08\,K_a}+K_T\right]}$$

The Ka is very large; therefore, the transfer function reduces to essentially $\frac{1}{K_T S}$, an integrating section. The equivalent block diagram reduces to the following:



Finally:

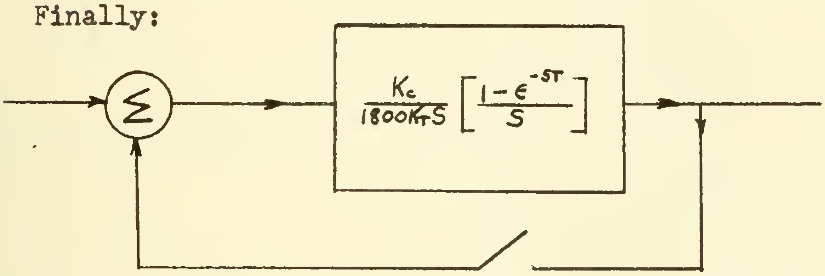


Fig. 18

Equivalent Block Diagram of Baldwin System

The equivalent gain can be expressed as $K_{eg} = \dfrac{K_c}{1800\ K_T}$.

The comparator gain K_c and the gear ratio are fixed. Therefore, the only variables in the system are essentially the value K_T, the tachometer feedback gain and the sampling rate $(f_s = \frac{1}{T})$.

By varying the K_T and the sampling rate it should be possible to obtain various degrees of response. The concern is first one of stability and its limit for the system with fixed values of K_T. The determining factor of stability is the sampling rate or pulses per unit time (f_s). The pulses are approximately 10 μsec wide and can be considered as impulses mathematically, $(\Delta t << T)$.

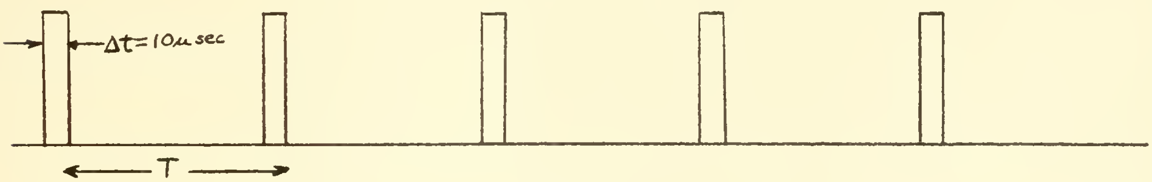


Fig. 19

Sample Pulses

$f_s = \frac{1}{T}$ where $1 \leq f_s \leq 100$ pulses/sec

The system transfer function is $F_o(S) = \dfrac{\frac{K_{eg}}{S}\left[\frac{1-\epsilon^{-sT}}{S}\right]}{1 + \frac{K_{eg}}{S}\left[\frac{1-\epsilon^{-sT}}{S}\right]}$

Stability Criteria

In order to establish the stability criteria it is necessary to equate the characteristic equation to zero.

$$1 + \frac{K_{eg}}{s} \frac{(1 - \epsilon^{-sT})}{s} = 0$$

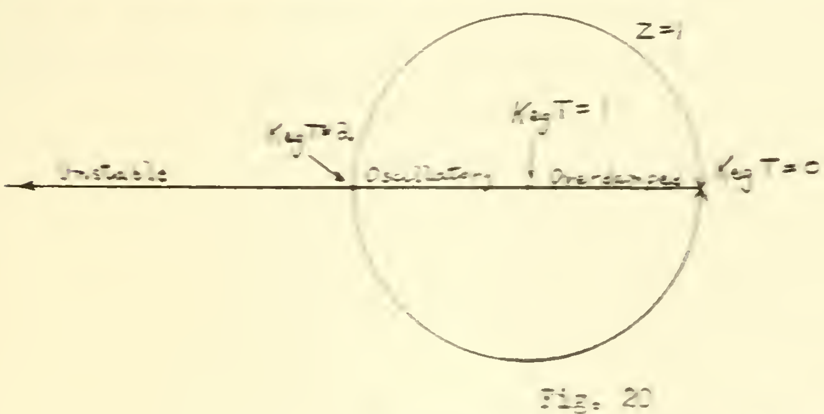
Since the system is of a sampled data type, the Z transform must be utilized. Expressing the characteristic equation in this form:

$$1 + K_{eg}(1-z^{-1})\frac{Tz^{-1}}{(1-z^{-1})^2} = 0 = 1 + \frac{K_{eg}Tz^{-1}}{(1-z^{-1})}$$

$$\text{or } 1 - z^{-1} + K_{eg}Tz^{-1} = 0$$

$$\therefore Z = 1 - K_{eg}T$$

Referring to the Z plane with a pole at Z = 1 and backlash not considered, the limit of stability can be estimated.



$$0 < K_{eg}T < 2$$

Fig. 20

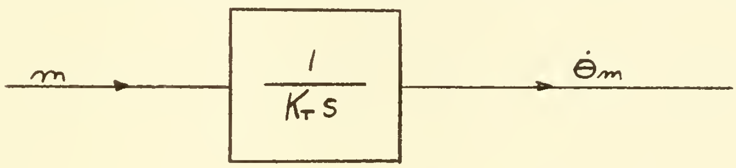
Z - Plane Plot of Baldwin System

For a finite settling time and no ripple $K_{eg}T = 1$. The limit of stability is $K_{eg}T = 2$, beyond this point oscillations are continuous. The limiting sampling rate therefore is $f_e = \frac{1}{T} = \frac{K_{eg}}{2}$.

When the comparator saturates the motor runs at a constant speed which depends on the control field and tachometer feedback voltages. As the tachometer feedback gain increases the motor reduces speed due to the increased torque. The increase in torque being a result of an increase of current in the tachometer circuit. Values of K_T versus motor speed are plotted in Fig. 21. The following analysis demonstrates the method of obtaining the various K_T values.

$\dot{\theta}_m$ = speed of motor in radians/sec

m = rms saturation voltage of the comparator (0.6 volts)



$$\dot{\theta}_m = \frac{m}{K_T}\left(\frac{1}{S}\right) = \frac{0.6}{K_T S}$$

$$|K_T| = \frac{0.6}{\dot{\theta}_m}$$

The minimum K_T occurs at the maximum speed of the servo motor, which is $\dot{\theta}_m$ = 3400 rpm or 356 rad/sec

K_T = .001685 volt/rad/sec

K_{eg} = 61.5 volt/rad/sec

Therefore, the highest theoretical limiting sampling rate is approximately f_L = 31 pps. Practically, the system was and never will be operated at this speed.

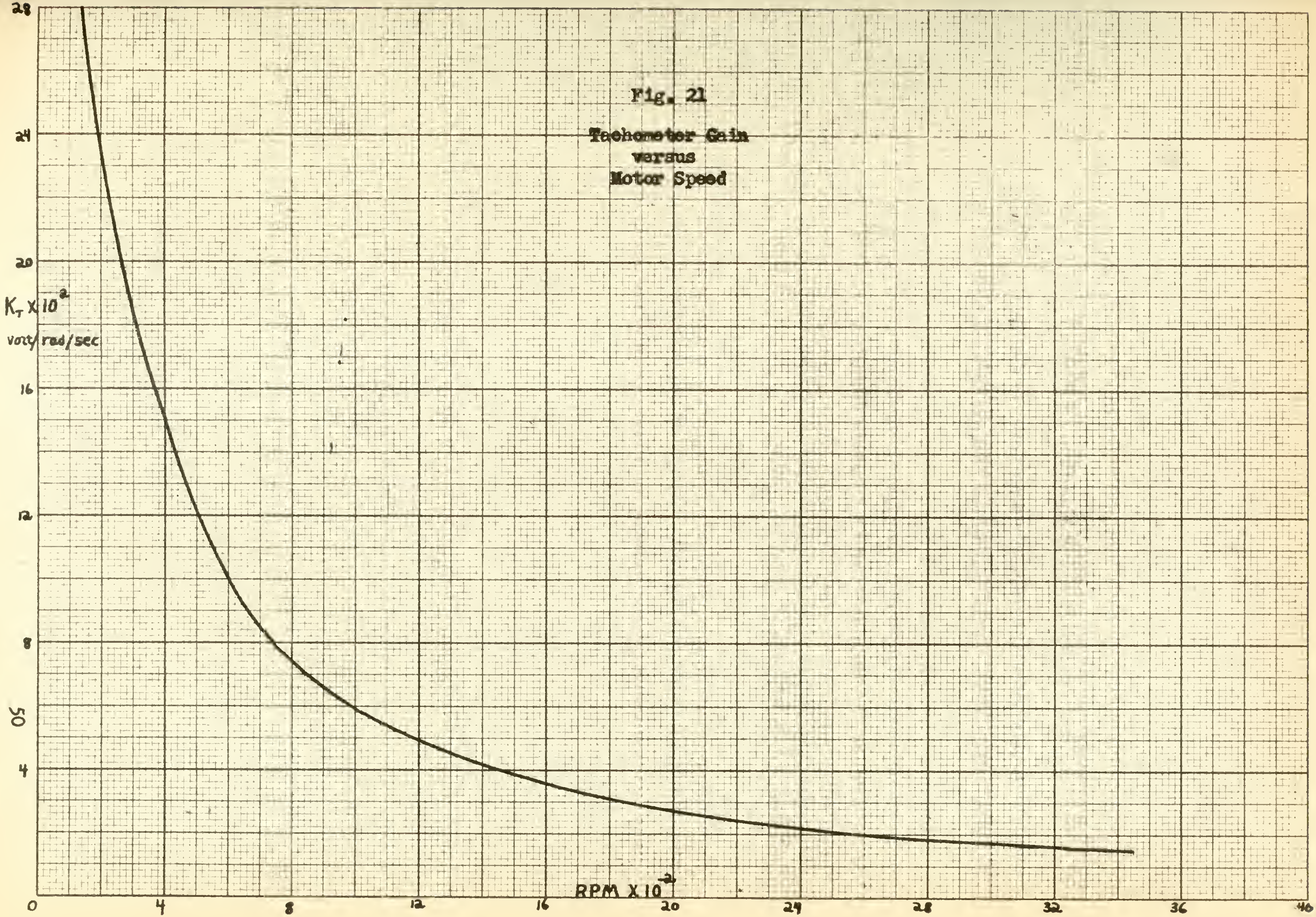


Fig. 21

Tachometer Gain
versus
Motor Speed

Originally, the control system as built by Philco WDL produced

$\dot{\theta}_m$ of 151.2 rad/sec (1450 rpm)

K_T = 0.00397 volt/rad/sec

K_{eg} = 26.0 volt/rad/sec

Therefore, the limiting sampling rate f_L = 13 pulses/sec.

In order to obtain lower sampling rates, the K_T value must be increased. This was accomplished by adding in parallel ever decreasing resistors to the tachometer feedback resistance. The original system was designed with two 330K ohm resistors in parallel or an equivalent resistance of 165K ohms.

Four values of tachometer resistance where used during tests to determine the error and response at various sampling rates.

Condition No. 1 Original System

Condition No. 2 Tachometer feedback resistance of 110K ohms

$\dot{\theta}_m$ = 90.6 rad/sec (866 rpm)

K_T = 0.00663 volt/rad/sec

K_{eg} = 15.6

f_L = 7.8 pulses/sec

Condition No. 3 Tachometer feedback resistance of 78.5K ohms

$\dot{\theta}_m$ = 73.1 rad/sec (700 rpm)

K_T = 0.0082 volt/rad/sec

K_{eg} = 12.6

f_L = 6.3 pulses/sec

Condition No. 4 Tachometer feedback resistance of 48K ohms

$$\dot{\theta}_m = 31.4 \text{ rad/sec (300 rpm)}$$

$$K_T = 0.0191 \text{ volt/rad/sec}$$

$$K_{eg} = 5.4$$

$$f_L = 2.7 \text{ pulses/sec}$$

Transient Response Below 2^5 Bits

It should be noted that the output quantity, θ_o, is a continuous function of time rather than a train of pulses. The purpose of this or any other system is to reproduce the command signal. In any sampled data system using a train of pulses the energy fed into the input by the train of pulses is less than would be fed in by the continuous, unsampled signal. Therefore the energy in the output of a system is less when the data is sampled for the same gain constant, K_{eg}, in the transfer function then when it is a continuous signal.

It is desirable to represent the pulse train as an impulse train because this simplifies the mathematical manipulations. This is permissable as long as the actual pulse is of short duration compared with the sampling period. By definition, a unit impulse has an infinite amplitude, however the pulse it represents has a finite amplitude. In impulse representation, the amplitude of the impulse remains infinite by definition, but the energy transferred by each impulse must be equal to that of the corresponding pulse.

The presence of the sampler is in itself destabilizing and it can be shown that it inserts an additional time lag. However, the system was made to operate more nearly like a continuous signal system by inserting

the zero order hold function of the comparator. This circuit forms an envelope by clamping the signal at the amplitude specified by the last pulse, and maintaining the signal amplitude until the next pulse is received.

A step by step transient analysis is easily performed on this type of circuit using step function response and the superposition principle. The system will operate as an open loop between samples, but the driving signal is of constant magnitude during this operation. The stepped signal may be represented mathematically as a series of step functions.

$$\mathcal{E}(t) = A_0 u(t) + (A_1 - A_0) u(t-T) + (A_2 - A_1) u(t-2T) + \cdots (A_n - A_{n-1}) u(t-nT)$$

Where $A_0, A_1, A_2, \text{----} A_n$ are the amplitude at the sampling instant and (T) is the sampling period. Note that the coefficients $(A_n - A_{n-1})$ may be either positive or negative.

The equation for the output variation is

$$\begin{aligned}\Theta_o(s) &= \frac{K_{eg}}{s}\mathcal{E}(s)\\ &= \frac{K_{eg}}{s}\left[\frac{A_0}{s} + \frac{A_1 - A_0}{s}\epsilon^{-Ts} + \frac{A_2 - A_1}{s}\epsilon^{-2Ts} + \cdots \frac{A_n - A_{n-1}}{s}\epsilon^{-nTs}\right]\end{aligned}$$

Taking the inverse transform

$$\theta_o(t) = K_{eg}\left[A_0 t\, u(t) + (A_1 - A_0) t\, u(t-T) + \cdots (A_n - A_{n-1}) t\, u(t-nT)\right]$$

It is apparent that the performance of the system is governed by the values K_{eg} and T. For fixed values of K_{eg}, increasing T causes increased displacement between samples, thus making the system more oscillatory. Decreasing T causes the system to approach a continuous or non-sampled type of system.

To illustrate these points, let $K_{eg} = 26$ (the original condition)

and f_s = 20, 25, 55.5, 100 samples/sec. Let the input to the system be a unit step displacement, then A_o = 1.0. The transient response is shown in Fig. 22.

It can be observed from this figure that at any sampling rate above 25 samples/sec the system, in theory, will not have any oscillations. Below this rate the system becomes more and more oscillatory until at 13 samples/sec instability is reached and there are continuous oscillations.

The result of decreasing the tachometer feedback resistance is to increase the rise time, settling time and to decrease the speed of the motor. However, it is now possible to use lower sampling rates. To illustrate these points, examine the second condition (see Fig. 23).

The equation for the output variation is

$$\theta_o(t) = 15.6\left[A_o t\, u(t) + (A_1 - A_o) t\, u(t-T) + \cdots + (A_n - A_{n-1}) t\, u(t-nT)\right]$$

As before, it is apparent that above a certain sampling rate (15.6pps) there will be no oscillations (in theory) and instability is at 7.8 samples/sec.

The transient response for the other two conditions is performed in the same manner, except that the response has a slower rise time and settling time.

Steady State Error

In a continuous system a type one system has a transfer function with one integration and a velocity lag error for a ramp input. The transfer function gain, K_{eg}, has the dimensions of inverse seconds and is the reciprocal of the steady state error if the ramp is a unit ramp. In the case of a sampled data system it is possible to define the steady state error coefficient as

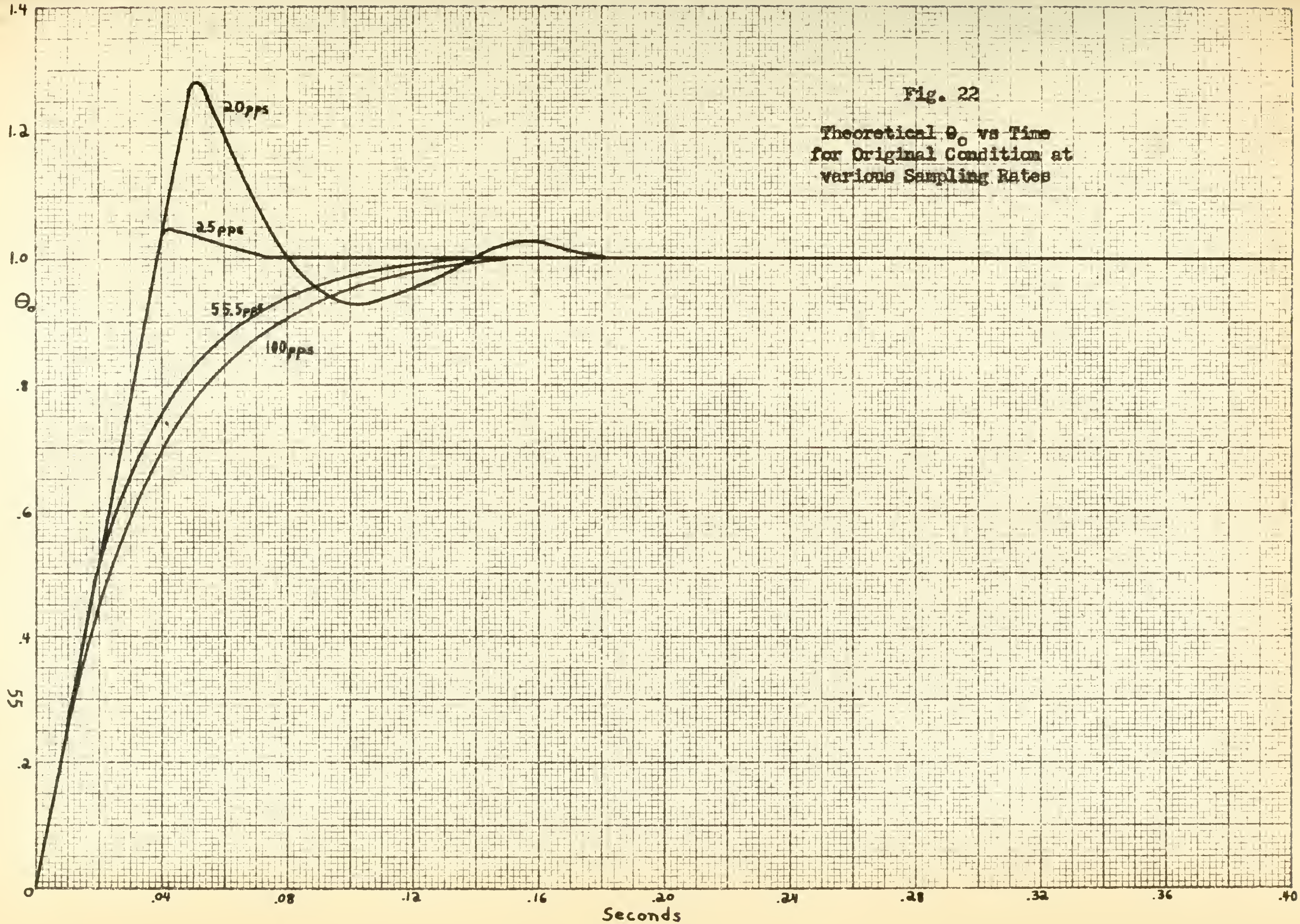


Fig. 22

Theoretical Θ_o vs Time for Original Condition at various Sampling Rates

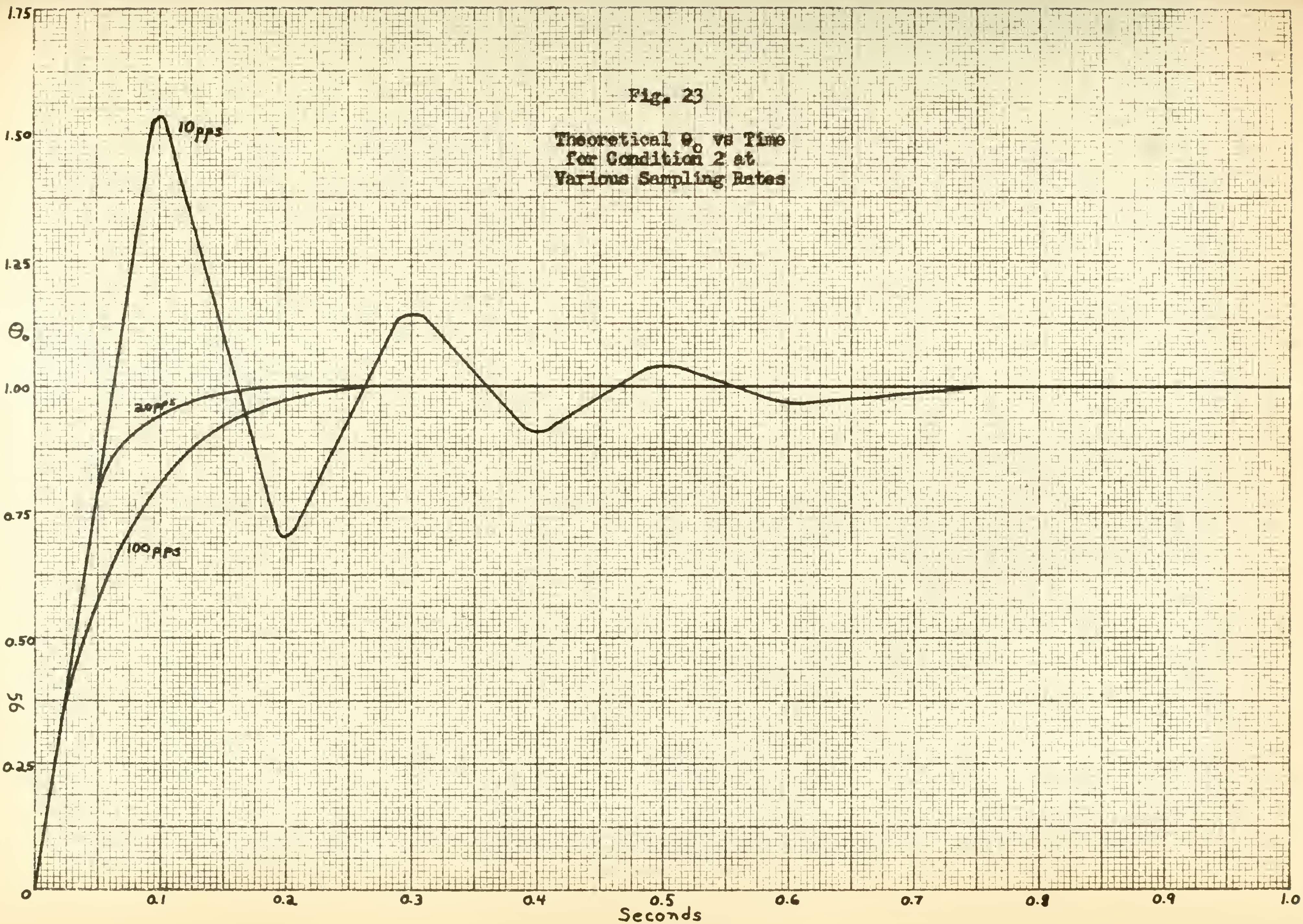


Fig. 23

Theoretical Θ_o vs Time for Condition 2 at Various Sampling Rates

$$K_v = \frac{1}{e_{ss}} = \frac{1}{T}\lim_{z \to 1}(z-1)\,G^*(z) \quad \text{where } G^*(z) = \frac{K_{eg} Z}{(Z-1)}$$

$$= \frac{1}{T}\lim_{z \to 1}(z-1)\frac{K_{eg} Z}{(Z-1)}$$

$$= \frac{K_{eg}}{T}$$

$$e_{ss}(\text{error at steady state}) = \frac{T}{K_{eg}} = \frac{1}{K_{eg} f_s}$$

As the gain of the system and the sampling rate in samples/sec is increased the steady state error will decrease. Therefore, as the tachometer feedback resistance is decreased the error will increase unless higher sampling rates are used. The variation of e_{ss} with the different gain conditions of the system versus the sampling rates are plotted in Fig. 24.

b. Datex Mechanical Encoder Analysis

The mechanical encoder replaced the Baldwin optical encoder in essentially the same system. (See Illus. 3 for the Datex System assembly). This was done in order to obtain uniformity in equipment, simplify the mathematics, and if possible to compare the two types of encoders. The only auxiliary equipment that the mechanical encoder required was the disc selector and a voltage reduction circuit from the selector to the translator.

To obtain encoder speeds (degrees/sec) near those used in the Baldwin system it was necessary to change the gear ratio. The Datex Encoder, Model CG-204, is actually a 17 bit encoder and the Philco Corporation had re-

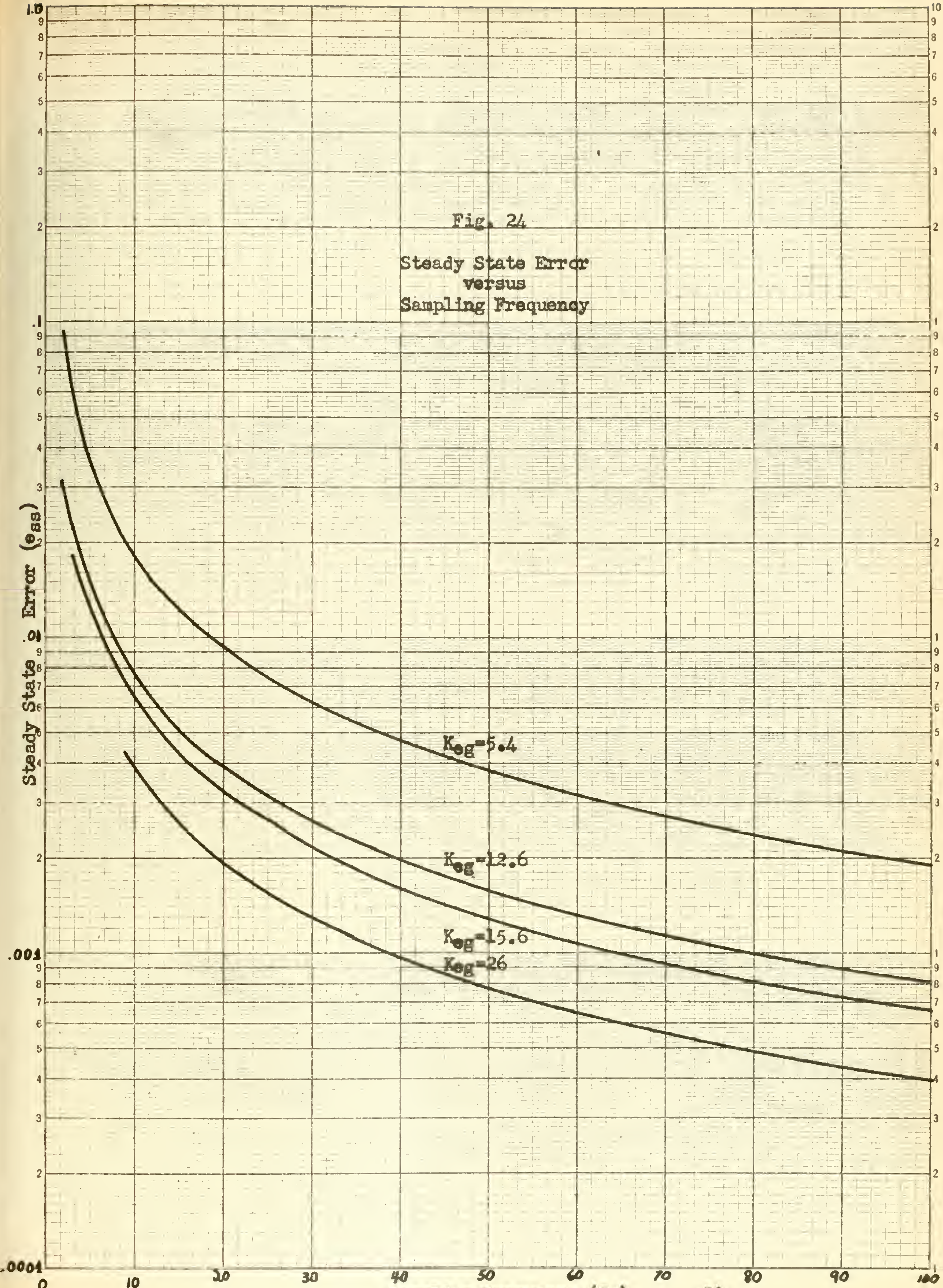


Fig. 24

Steady State Error
versus
Sampling Frequency

quested a 16 bit encoder. To conform with this request, Datex merely clipped off the brush from the most significant track. The resultant most significant track was only 90° of the circumference of the rotating disk, compared with the usual 180°, necessitating the reduction of the gear ratio by one half of its former value to 900/1.

By careful construction of the gear train between the motor and the encoder the backlash was reduced to approximately $\pm$ 1 bit. This eliminated an important non-linearity that was present in the Baldwin System.

From the preceeding discussion, it is obvious that the transfer functions are basically the same except for the reduction of the gear ratio. The absence of a sampler in the encoder reduces the mathematics considerably. Since the system now has a continuous feedback input to the comparator, instability is practically eliminated. The block diagram appears as follows:

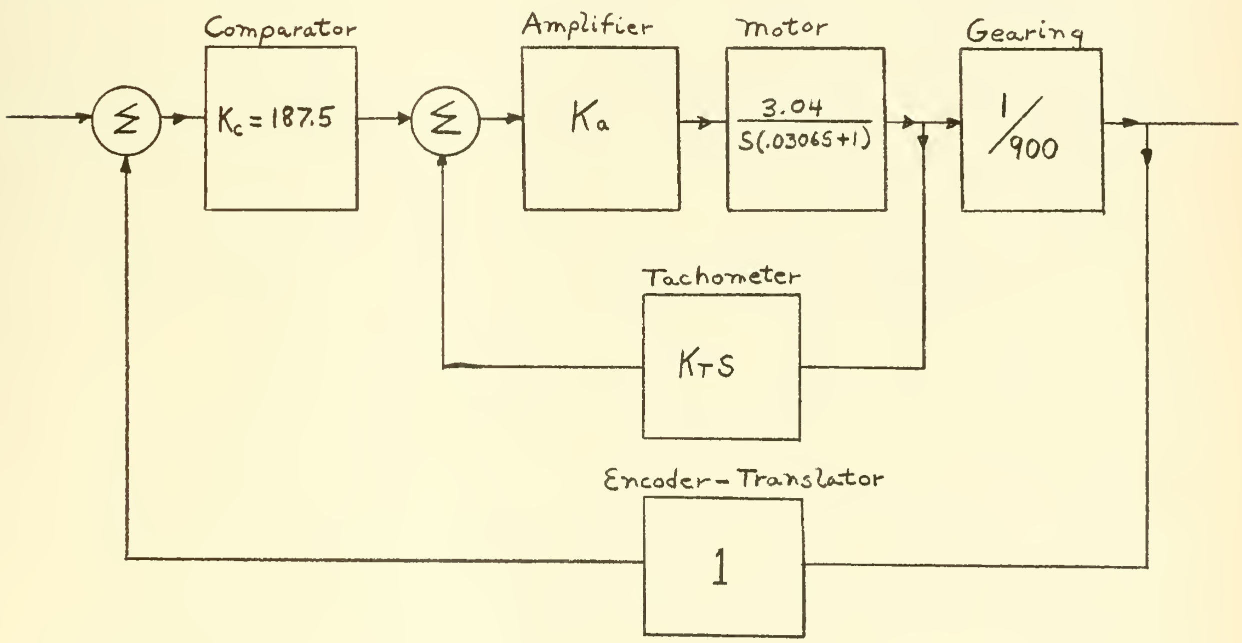


Fig. 25

Datex Encoder Servo System

As in the Baldwin Analysis the K_a is large and the system reduces to:

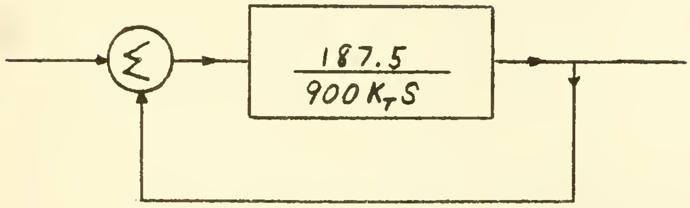


Fig. 26

Equivalent Datex System

The closed loop transfer function equals:

$$\frac{\frac{187.5}{900 K_T}}{S + \frac{187.5}{900 K_T}}$$

For a step input of 32 bits or less the response will be of exponential form. The output versus time is easily calculated.

$$\theta_i(t) = A\,u(t)$$

$$\theta_o(t) = A\left(1 - \epsilon^{-K_{eg}t}\right)u(t) \quad \text{where } K_{eg} = \frac{187.5}{900 K_T}$$

The only variable in this system is the tachometer feedback resistance, K_T. For illustration, three values of K_T are employed in the identical manner as in the Baldwin Analysis.

Condition No. 1 Original Tachometer Setting (165K ohms)

$\dot{\theta}_m$ = 182.5 rad/sec (1745 rpm)

K_T = .00328 volt/rad/sec

K_{eg} = 63.5

$\theta_o(t) = A(1 - \epsilon^{-63.5t})\ u(t)$

Rate of Encoder = 5.8°/sec

Condition No. 2 Tachometer Feedback Resistance of 78.5K ohms

$$\dot{\theta}_m = 104.6 \text{ rad/sec (1000 rpm)}$$

$$K_T = 0.00574 \text{ volt/rad/sec}$$

$$K_{eg} = 36.3$$

$$\theta_o(t) = A(1-\epsilon^{-36.3t}) \quad u(t)$$

Rate of Encoder = 3.33°/sec

Condition No. 3 Tachometer Feedback Resistance of 48K ohms

$$\dot{\theta}_m = 62.8 \text{ rad/sec (600 rpm)}$$

$$K_T = 0.00955 \text{ volts/rad/sec}$$

$$K_{eg} = 21.8$$

$$\theta_o(t) = A(1-\epsilon^{-21.8t}) \quad u(t)$$

Rate of Encoder = 2°/sec

The transient response, $\theta_o(t)$, for the three conditions is plotted in Fig. 27. These are exponential type responses and theoretically the system could never become unstable.

The steady state error for the three conditions is simply the inverse of K_{eg}: 1) .01575 rad

2) .0275 rad

3) .0459 rad

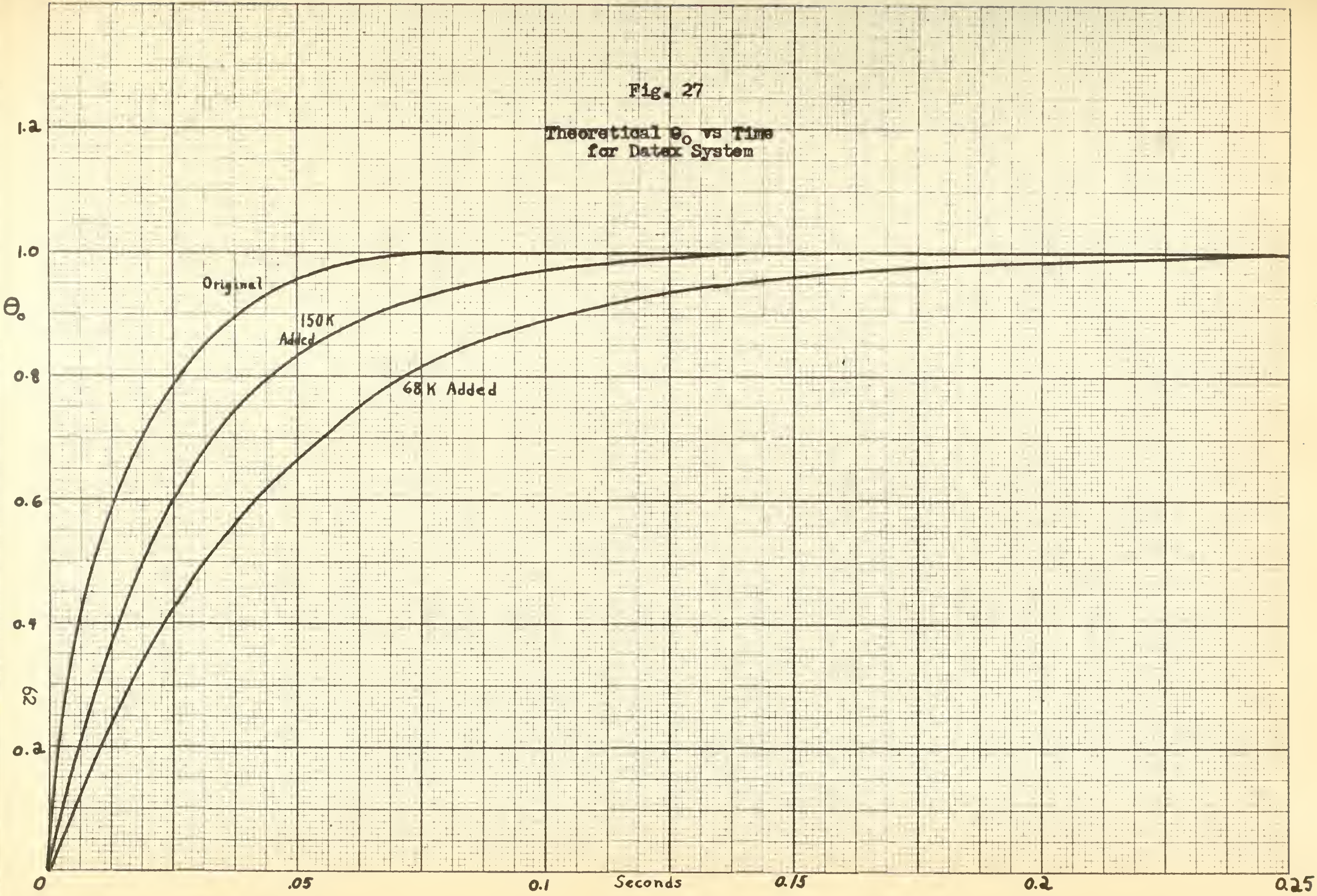


Fig. 27

Theoretical Θ_o vs Time for Datex System

5. DISCUSSION AND RESULTS

Testing Procedures

The Baldwin encoder was originally installed in the control system designed and built by the Philco engineers. Since the encoder is a sampler, it is important to determine the effect on the system of varying the sampling rate. It was necessary to modify the Baldwin programmer so that any sampling rate between 1 and 100 pulses per second could be applied. Modification was performed by conversion from the five position switch to a potentiometer. The measurement of any particular sampling rate was accomplished by connecting a Tektronix 535 oscilloscope to the programmer and determining the period between pulses.

Philco's consultant Dr. G. F. Franklin, had recommended that the system be designed to use a sampling rate of 25 pulses per second. At this rate the control system would act essentially as a continuous system and provide sufficient time spacing in the time sharing of the computer with the other two types of systems. Generally, a tracking system could have three types of controlled movement: elevation, azimuth and traverse. Thus, during the tests a selected sampling rate was 25 pulses per second.

The following sampling rates were used, in conjunction with each of the four conditions mentioned in the analysis: 92.5, 70, 55.5, 40, 30, 25, 15, 10, 7, 5, 4, 3, 2, 1 samples per second.

Commencing with the maximum sampling rate, the following effects on the system were to be determined:

1. Final average error for various step inputs,
2. Effect of tachometer feedback voltage variation on
    a. number and magnitude of oscillations
    b. speed of motor

c. servo amplifier gain

3. Effect of Servo Amplifier gain variation upon

a. number and magnitude of oscillations

b. speed of motor

c. tachometer feedback voltage

4. The limiting stable sampling rate,

5. Variation of strobe light voltage upon accuracy and control,

6. Noise level and its variation with sampling rate,

7. Speed of motor at any one condition.

Runs were made for four values of tachometer feedback resistance: 165K ohms, 110K ohms, 78.5K ohms, 68K ohms and with the tachometer feedback removed. For each of these, the tachometer and amplifier gain potentiometer were adjusted so as to give optimum system response. The tach and gain voltages for these settings were then recorded. Error signals were then generated by closing a combination of switches in the upper row of sixteen switches (one for each bit) as shown in Illus. 2. System accuracy response to these inputs were then measured by noting the correlation between the switch combination used and the steady-state light combination produced at the translator. This value was recorded for each switch position.

Outputs from the comparator, servo amplifier and tachometer feedback voltage before being chopped, were recorded by use of a multi-channel Brush recorder capable of speeds up to 250 mm per second. The error signal from the comparator provides direct plotting of the transient response since,

$$E(t) = \theta_i(t) - \theta_o(t) \quad \text{or} \quad E(t) = 1 - \theta_o(t) \quad \text{for a unit step}$$

input. The speed of the motor can be approximated from the dc tachometer

voltage, using the relationship for the 2 phase Diehl motor that speed (rpm) = $\frac{V_T \text{ (volts) } 1000\text{rpm}}{65 \text{ volts}}$. In addition, the encoder speed was timed with a stop watch and multiplied by the gear ratio to give a precise motor speed. The amplifier output provides a measure of the control field voltage to the motor and indicates time delays between application of the input and the response of the comparator and amplifier.

Datex Encoder. In order to permit some degree of comparison between the two types of 16 digit encoders, the Datex system was set up to utilize the same motor, translator, tachometer feedback, amplifier, and comparator as was used with the Baldwin. Since requirements other than those of this project called for inter-changeable use of the two encoders in the system, it was decided to include a switching relay arrangement in the overall circuit. This required sixteen relays, one for each digit output of the Baldwin or Datex, and was placed in the encoder output circuit prior to the translator. This relay connection is shown in Illus. 3.

Since the Datex output is continuous while that of the Baldwin is sampled it was necessary to by-pass the flip-flop circuits of the translator boards with this input. This was accomplished by using separate wiring from the relay board.

The test procedures used were similar to that of the Baldwin system with continuous output only. The comparator, amplifier, and tachometer feedback output voltage variations were recorded on the multi-channel Brush recorder. As in the Baldwin System the transient response could be plotted from these recordings.

It was desired to determine the following effects on the Datex system as listed for the Baldwin System: 1, 2, 3, 6. Three values of tachometer

feedback resistance were used: 165K ohms, 110K ohms and 68K ohms. A set of readings was also taken with the tachometer feedback circuit disconnected. For each of these the tachometer and amplifier gain potentiometer were set so as to give the same corresponding voltages used in the Baldwin system. The remainder of the Datex procedure was also identical.

Results and Discussion

Each control system would not function if the amplifier gain potentiometer was rotated more than approximately three-eighths of its range movement. Beyond this point jitter was excessive. Jitter is unwanted noise of a predominantly oscillatory nature; it can be wideband or can contain only one or two frequencies. Various nonlinear elements and system generated noises can cause or contribute to jitter in electromechanical servos. The possible system non-linearities include coulomb friction between the motor shaft and its bearings, the gear-compliance characteristic including backlash, quantizing effect of the encoder and the saturating characteristic of the servo amplifier. Other causes of jitter include limiting, dead space, granularity effects, d-c static unbalance voltage, amplifier pickup noise and vibrator noise.

There was a great amount of backlash in the Baldwin system and little or none in the Datex system; therefore backlash was not the major cause since jitter was present at the same point in both systems. Coulomb friction was not an important cause since it was relatively small.

From previously mentioned Brush recordings, there was a noticeable continuous unbalanced oscillation of approximately 60 cps which was not large enough to rotate the motor when the system was being operated without jitter and no signal input. A d-c static unbalance voltage between

the halves of the push-pull output state of the amplifier could combine with the 60 cycle reference voltage and produce a strong 60 cycle motor torque. Although the 60 cycle torque is attenuated by the motor load dynamics, the servo output can contain a 60 cycle oscillation even in the presence of zero input signal. However, this oscillation indicated by the recording pen did not have the amplitude necessary to overcome coulomb friction. When the amplifier gain potentiometer was adjusted beyond a certain critical point, the amplitude was sufficiently high and produced the jitter.

There are, in this case, two other possibilities which could have caused the jitter and the small 60 cps unbalanced oscillation by themselves or in combination with the d-c unbalance of the amplifier output stage. Amplifier pickup noise (primarily line frequency) will cause a voltage in the amplifier output which can be appreciable in this type of high gain servo amplifier. Since the amplifier output is applied to the motor control winding, the motor will oscillate in response to this control winding voltage. If the amplifier pickup noise is severe, it will cause jitter in the output. Also, the synchronous vibrator or chopper in the tachometer feedback section can produce noises of frequencies determined by the chopper reed driving frequency (60cps) and mechanical resonance can cause important output noise when the impedance of the driving voltage source is high and the amplifier gain is large.

The actual jitter frequency was measured to be 60 cps by a strobe light. A recording of the chopped tachometer feedback voltage indicated appreciable noise. From all indications, it appears as if the chopper noise was the major cause of jitter; however, this does not eliminate the

possibility of amplifier pickup noise and/or d-c static unbalance of the amplifier output stage as a cause. Jitter caused by chopper noise can be reduced by obtaining the vibrator driving voltage from a low impedance source such as a filament transformer.

The test runs were performed on the Baldwin system at each of the four conditions at the sampling rates mentioned in the procedure. For each rate a final average error was recorded and the result plotted in Fig. 28. Referring to this figure, it can be seen that the error, for each condition, is fairly constant until the limiting or unstable sampling rate is approached even though the oscillations and/or amplitudes increase as the sampling rate decreases. The error increases rapidly just before the unstable sampling rate as shown on the left side of the figure. However, the system is limited to $\pm$ 32 bits of error and it will oscillate between these limits at the unstable sampling rate.

Since it was necessary to reduce the tachometer feedback resistance to obtain the lower sampling rates, the effective overall gain and velocity of the motor were reduced. Thus, the effective coulomb friction was increased causing a larger final error. This error could be reduced by increasing the amplifier gain adjustment just enough to produce a small jitter which tended to overcome the increased coulomb friction. It was possible to reduce the error to $\pm$ 1 bit below 10 pps. In a large commercial servo system a separate 60 cycle dither could be applied to a large motor shaft to reduce the error caused by the presence of large coulomb friction.

The limiting sampling rates were recorded and compared with the theory from the analysis (see Table I). Considering that nonlinearities such as

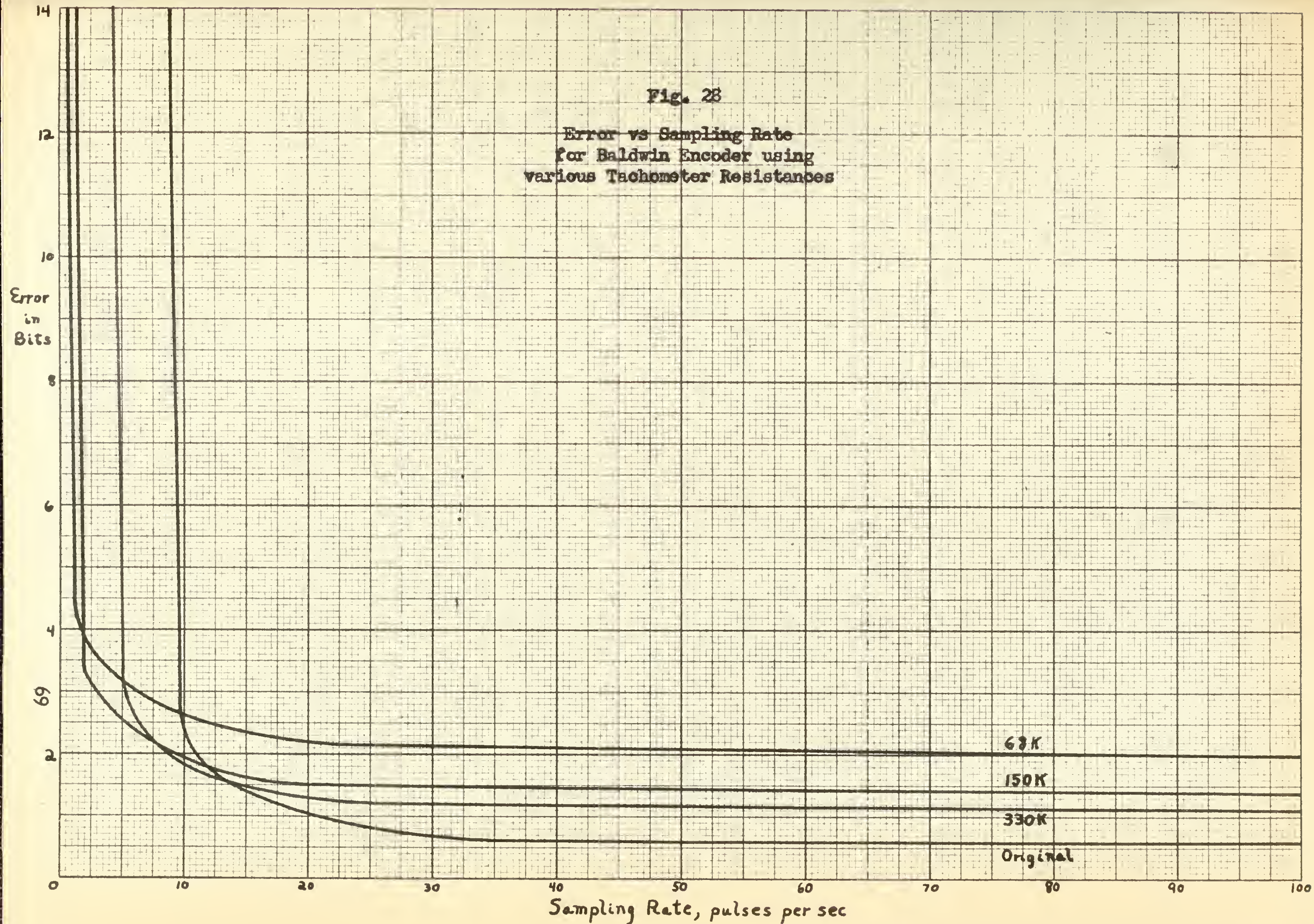


Fig. 28

Error vs Sampling Rate
for Baldwin Encoder using
various Tachometer Resistances

coulomb friction were neglected, the results check reasonably well.

Condition	Theory	Actual
Original	13	10
2	7.8	7
3	6.3	5
4	2.7	2

Table I

Baldwin System Limiting Sampling Rates for Stability

Reducing the strobe light voltage in the programmer did not reduce the accuracy until a critical level of approximately 600 volts was reached. The photocells did not receive sufficient illumination at this point and the system single phased, rotating in one direction continuously.

Referring to Fig. 29, the effect of varying the amplifier and tachometer feedback potentiometers on the system can be observed. The Datex system was utilized for this test since it provided smoother curves. As might be expected, reducing the amplifier gain slightly increases the overshoot and reducing the tachometer feedback permits faster response (K_T is smaller) but the saturation effects are not as damped and therefore increased oscillations and oscillation amplitudes result.

As the sampling rate decreases the noise level voltage (peak to peak) remained constant. However, it was noticed that the noise density decreased.

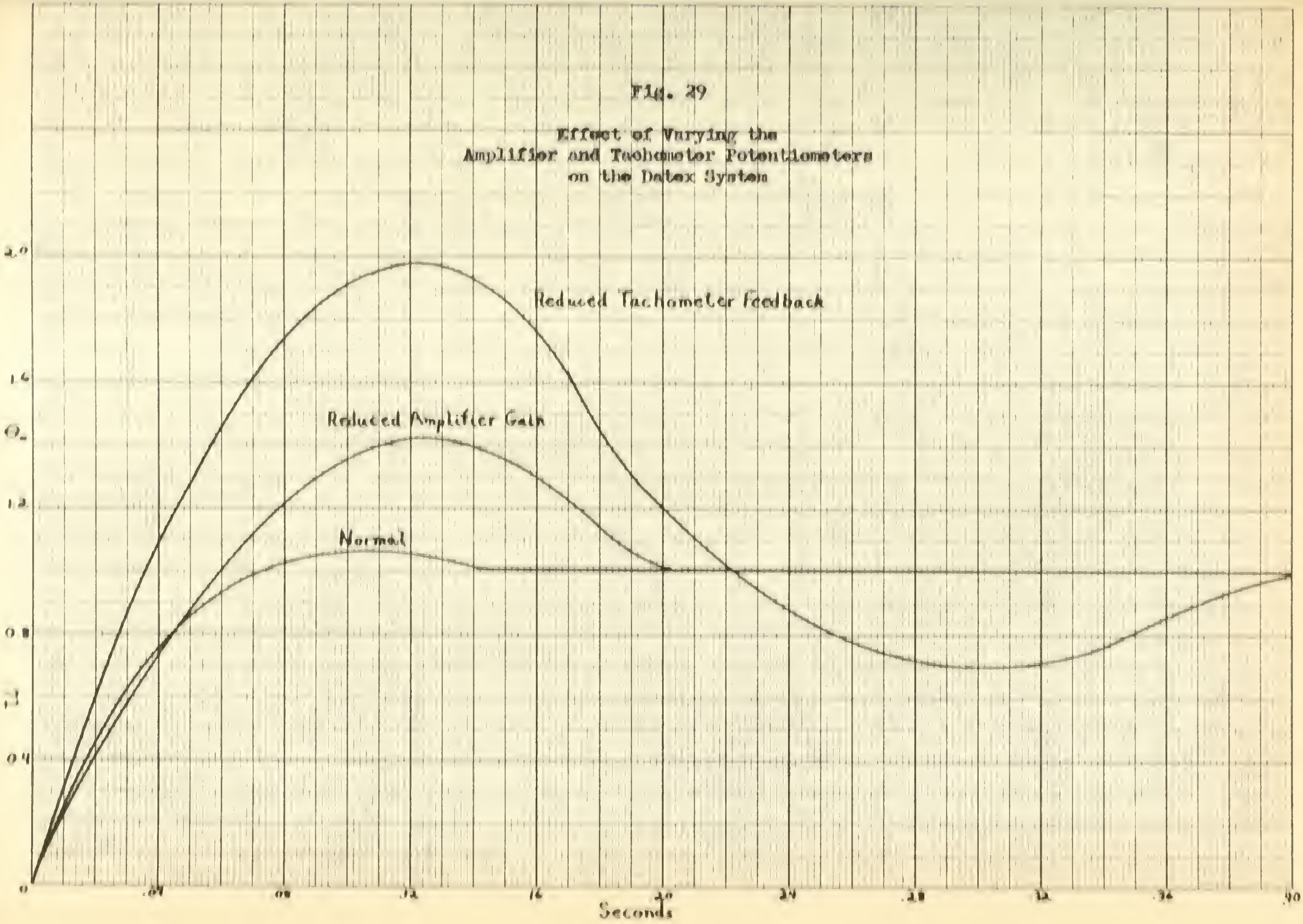


Fig. 29

Effect of Varying the
Amplifier and Tachometer Potentiometers
on the Datex System

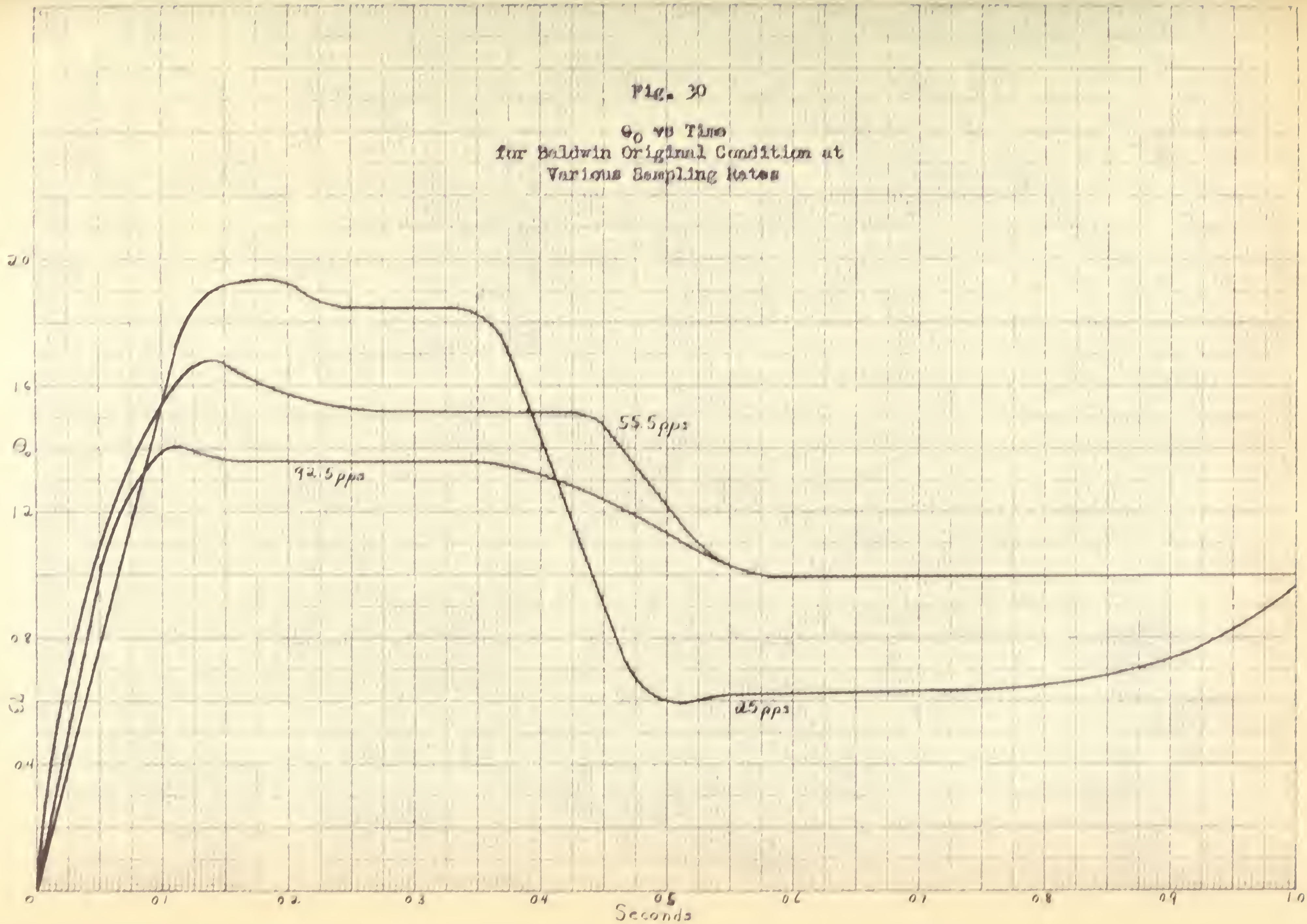
Transient Response of Baldwin System

The actual transient response curves for the 32 bit null area and under the first two conditions of the Baldwin system are plotted in Figs. 30 and 31. In order to compare the actual curves with the theoretical curves, both sets of curves are replotted in Figs. 32 through 37. Referring to these figures, it can be observed that the backlash present in the Baldwin system has an important effect on the response. The relatively flat area on the top and bottom of the actual curves can be attributed to the excessive backlash present. The rise time in each case agrees closely which indicates correlation between the mathematical analysis and the actual conditions. However, the actual curves have a large overshoot which increased with decreasing sampling rate. This is a result of neglecting the inertia of the load in the theory. The Baldwin encoder disc is rather heavy, presenting a moment of inertia of 18.2 lb-in^2. It required 8-16 oz-in of breakaway torque or an average of 10 oz-in of running torque. In contrast, the Datex encoder had a breakaway torque of 0.6 to 1.0 oz-in and an average running torque of 0.7 oz-in. The moment of inertia was 0.15 lb-in^2. Thus, inertia has an effect in the Baldwin system which can not be neglected. In the Datex system inertia will not have an appreciable effect when compared to the first system. This is confirmed by referring to the presence or absence of an overshoot in the three actual sets of curves (Fig. 38).

The slight curve at the top of the actual Baldwin transient response is caused by the springing effect of the various shafts from the motor through the gear train and the encoder disc.

Fig. 30

θ_0 vs Time
for Baldwin Original Condition at
Various Sampling Rates



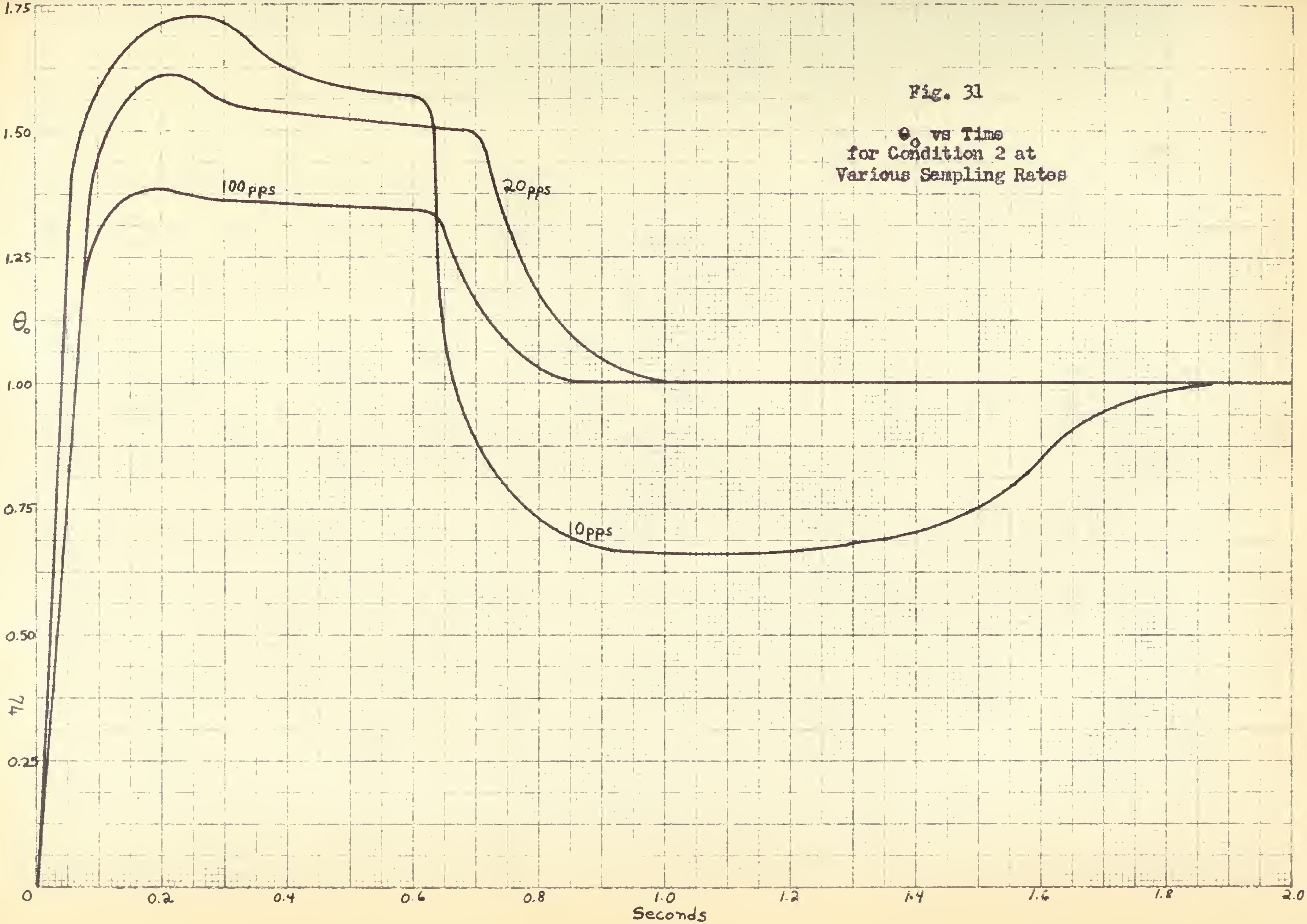


Fig. 31

θ_o vs Time for Condition 2 at Various Sampling Rates

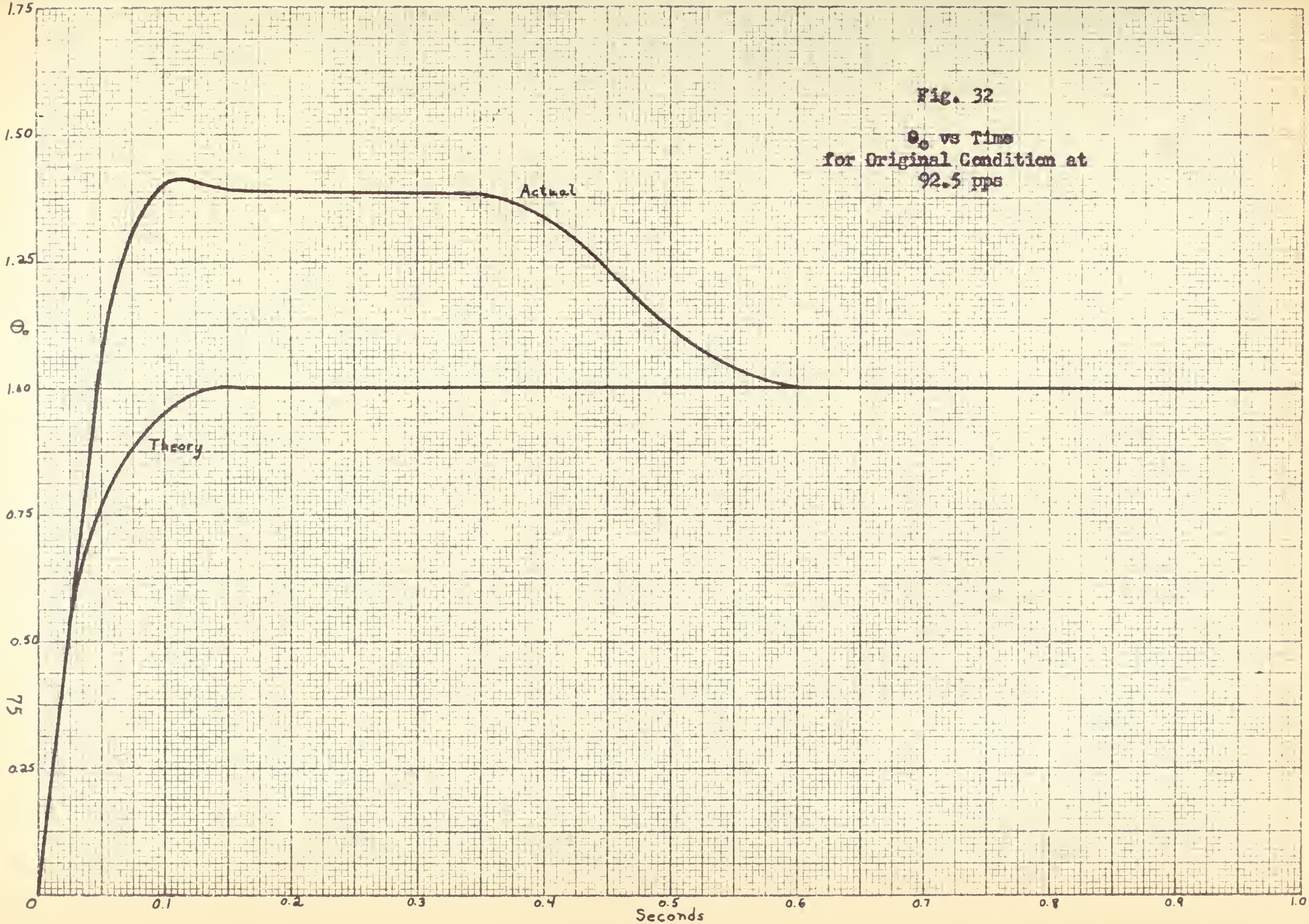


Fig. 32

Θ_o vs Time
for Original Condition at
92.5 pps

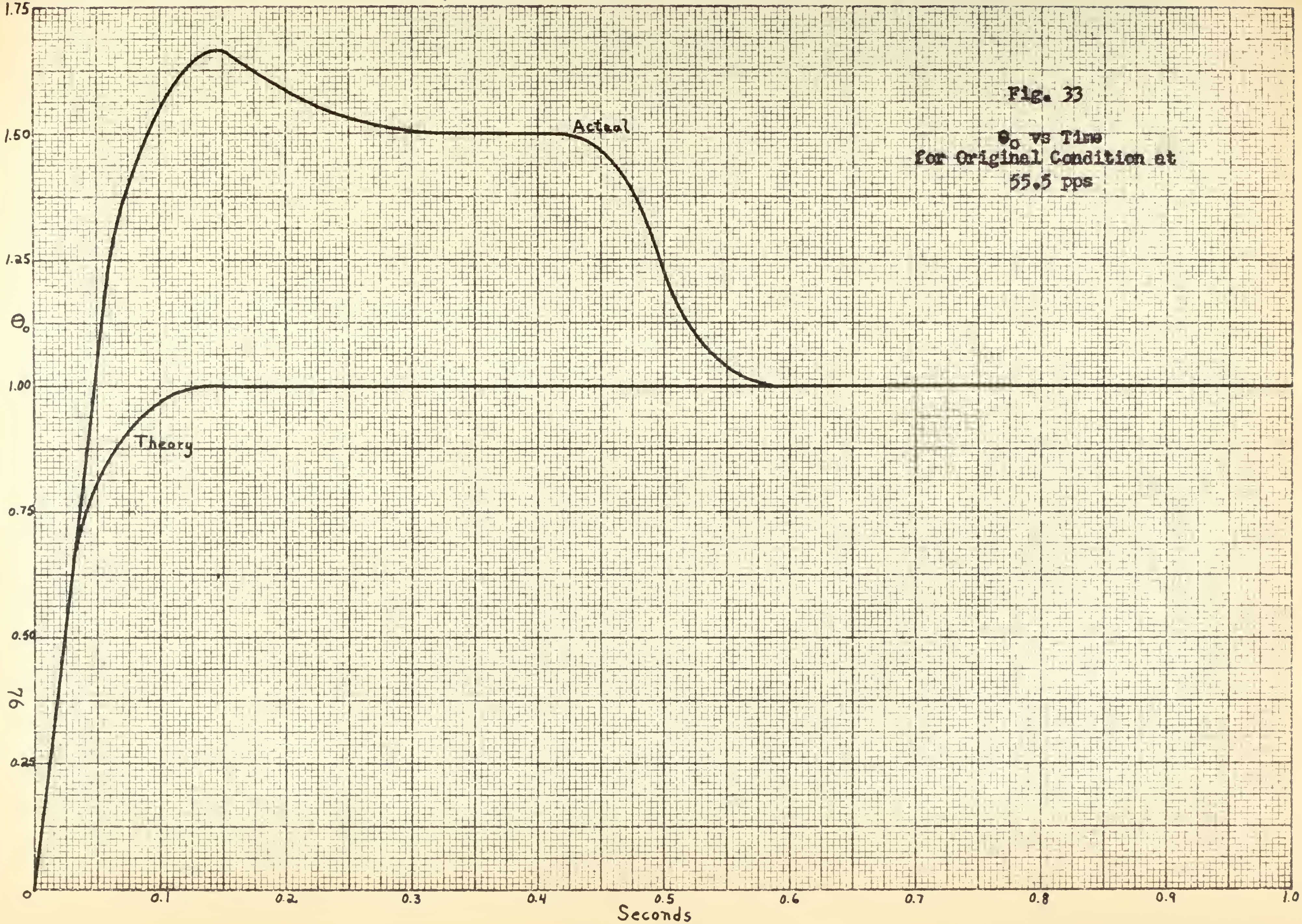


Fig. 33

Θ_o vs Time
for Original Condition at
55.5 pps

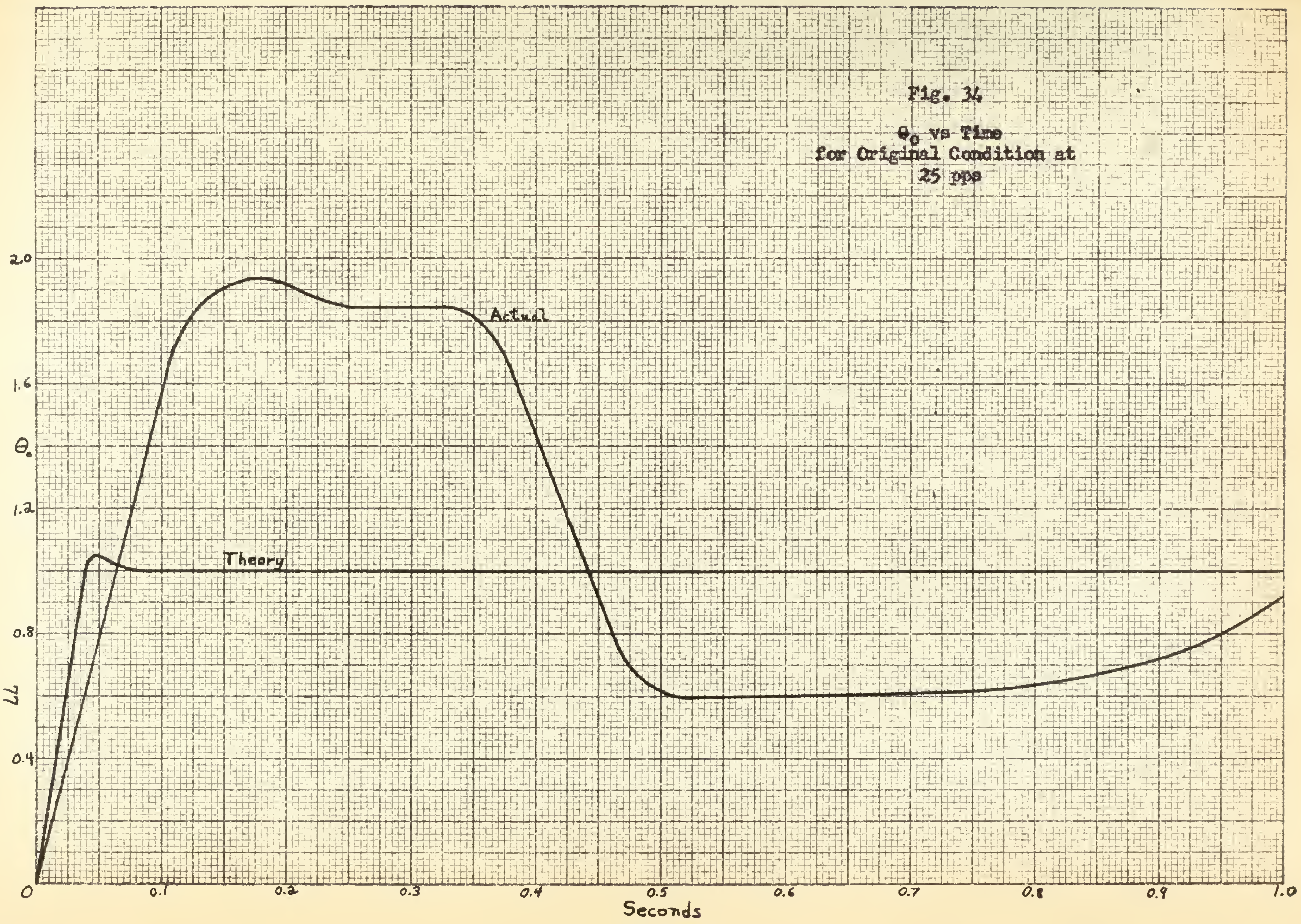


Fig. 34

Θ_o vs Time
for Original Condition at
25 pps

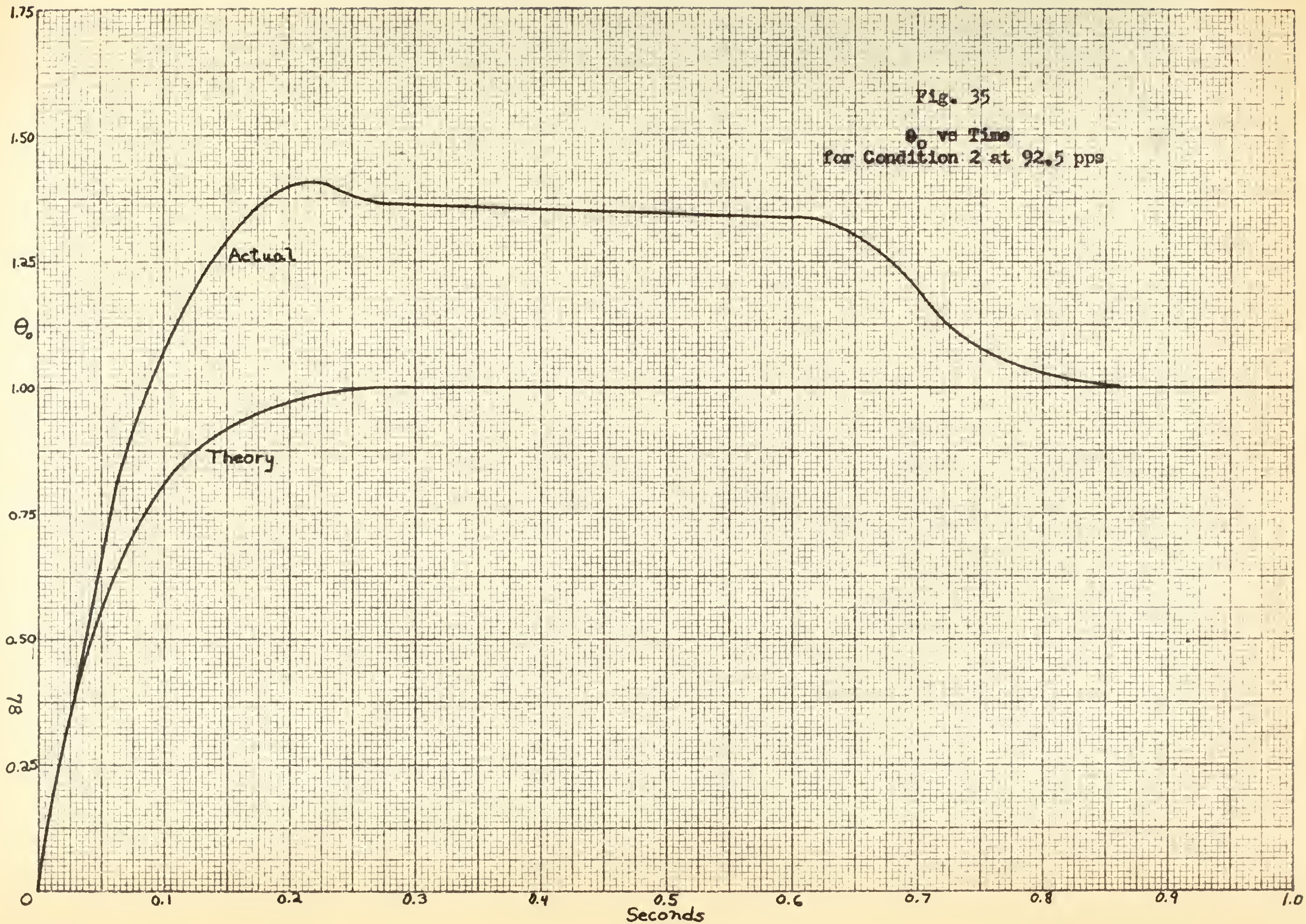


Fig. 35

Θ_o vs Time
for Condition 2 at 92.5 pps

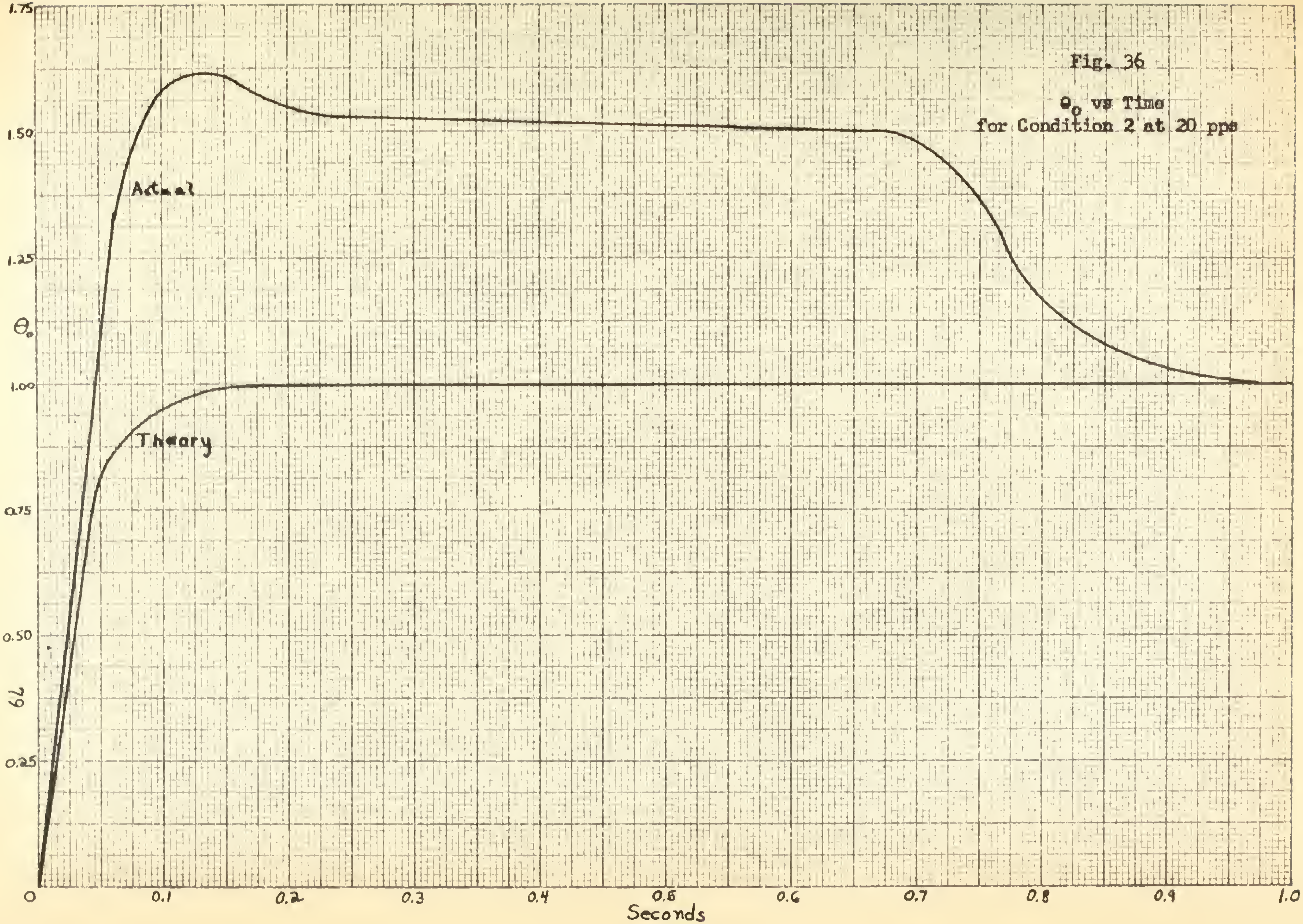


Fig. 36
Θ_o vs Time
for Condition 2 at 20 pps
Actual
Theory
1.75
1.50
1.25
Θ_o
1.00
0.75
0.50
0.25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Seconds

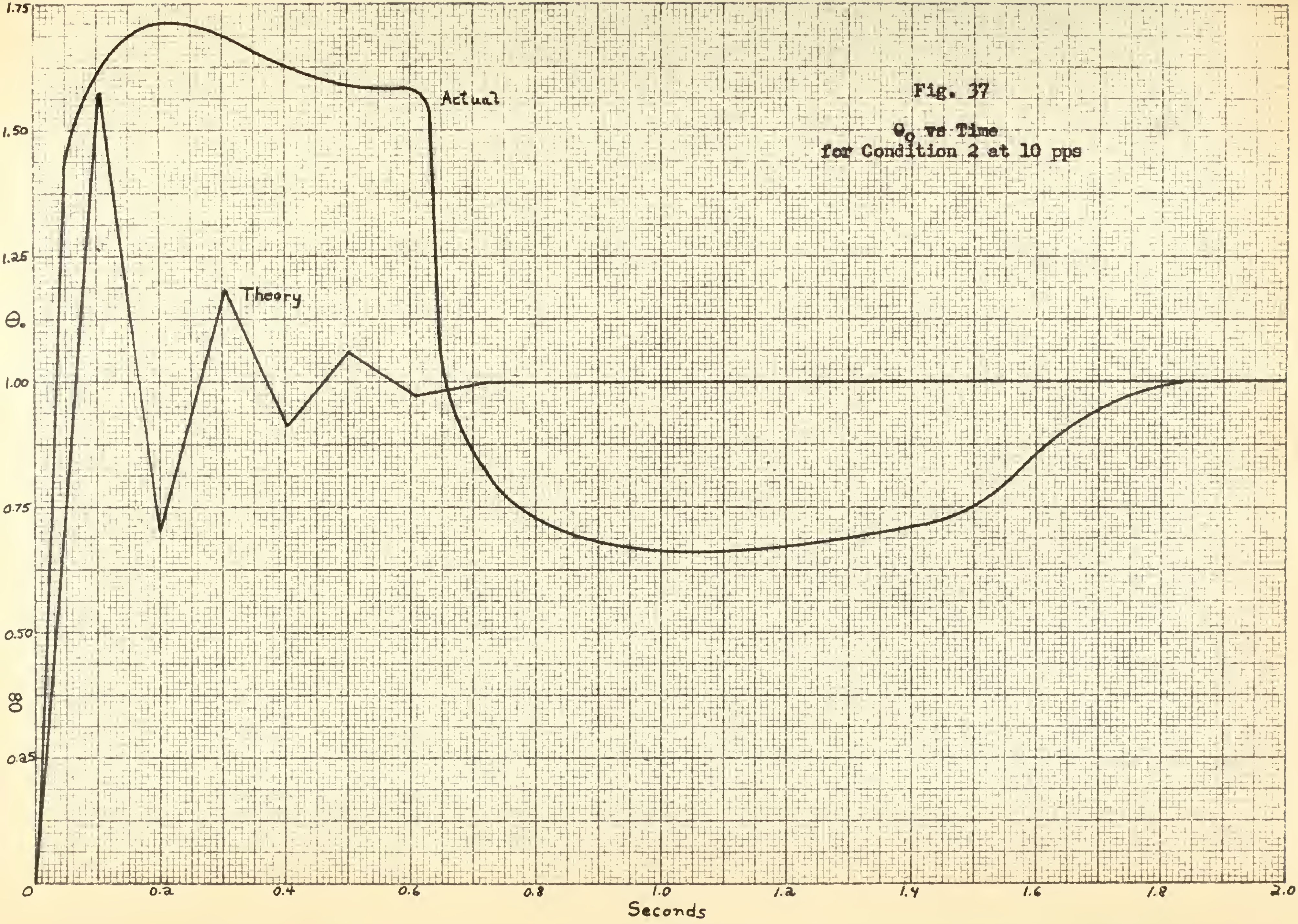


Fig. 37

Θ_o vs Time
for Condition 2 at 10 pps

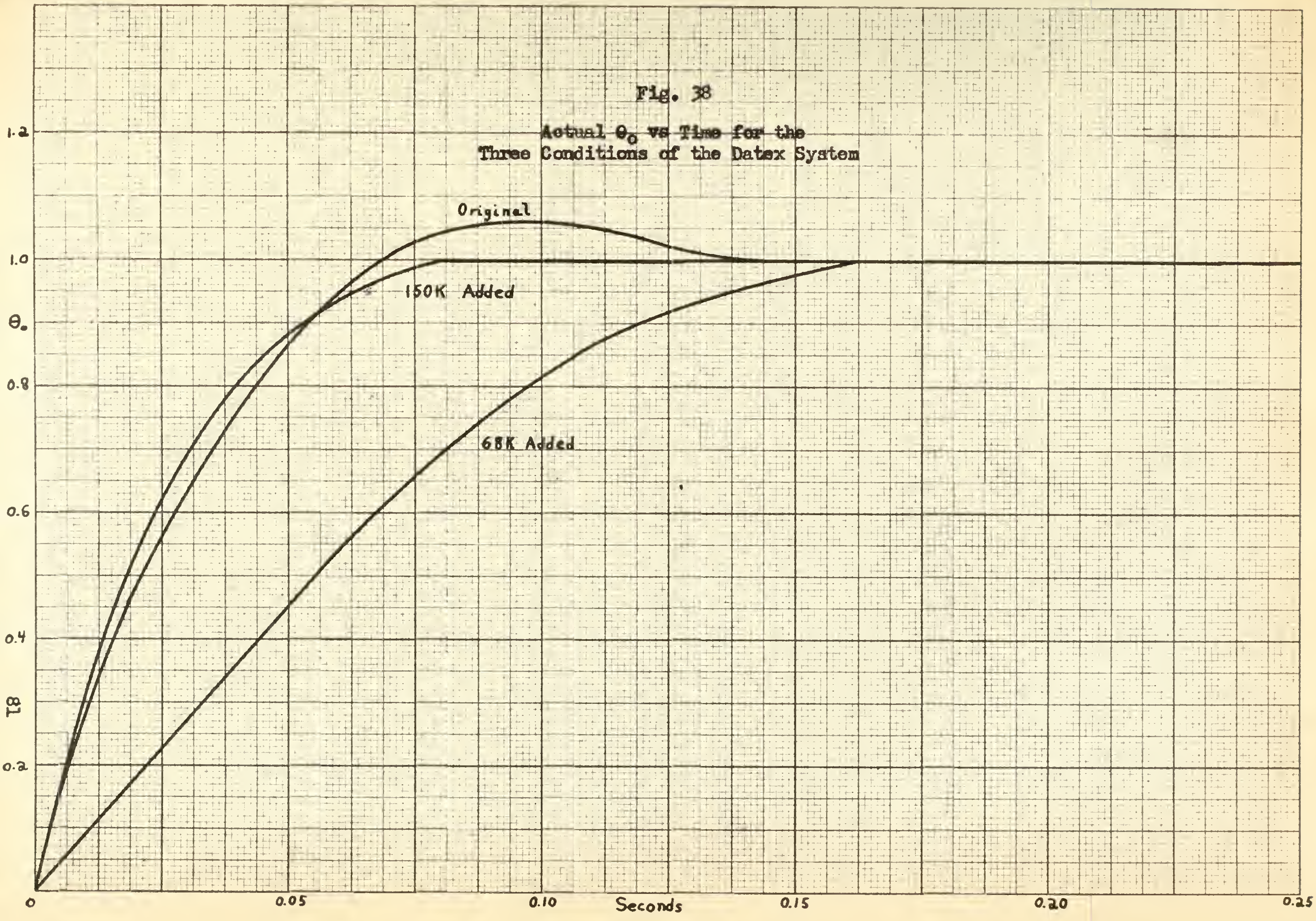


Fig. 38

Actual θ_o vs Time for the Three Conditions of the Datex System

Transient Response of Datex System

Figs. 39 through 41 are comparison plots of the transient response for the Datex system. Correlation between theory and actual results is noted to be very close with this encoder. In this continuous system with a minimum of backlash in the gear train (of the order of one or two bits in 65,536) this improvement in response over that of the Baldwin system was to be expected.

The slight overshoot of the system indicated in Fig. 39, of the order of about 5%, is due to some mechanical inertia not considered in the theoretical calculations. This is seen to occur only at the high speed tachometer resistance combination. Even then, settling time is increased by only about 15% (approximately 0.02 seconds).

Rise time comparison shows close agreement between actual and theory also. This is especially true of the higher speed-of-response runs (higher tachometer circuit resistance). The greater separation between the two curves of Fig. 41 can be attributed to the fact that the actual resistance combination used was closer to 67K and also to difficulties in accurately reading the brush pen recording for this run.

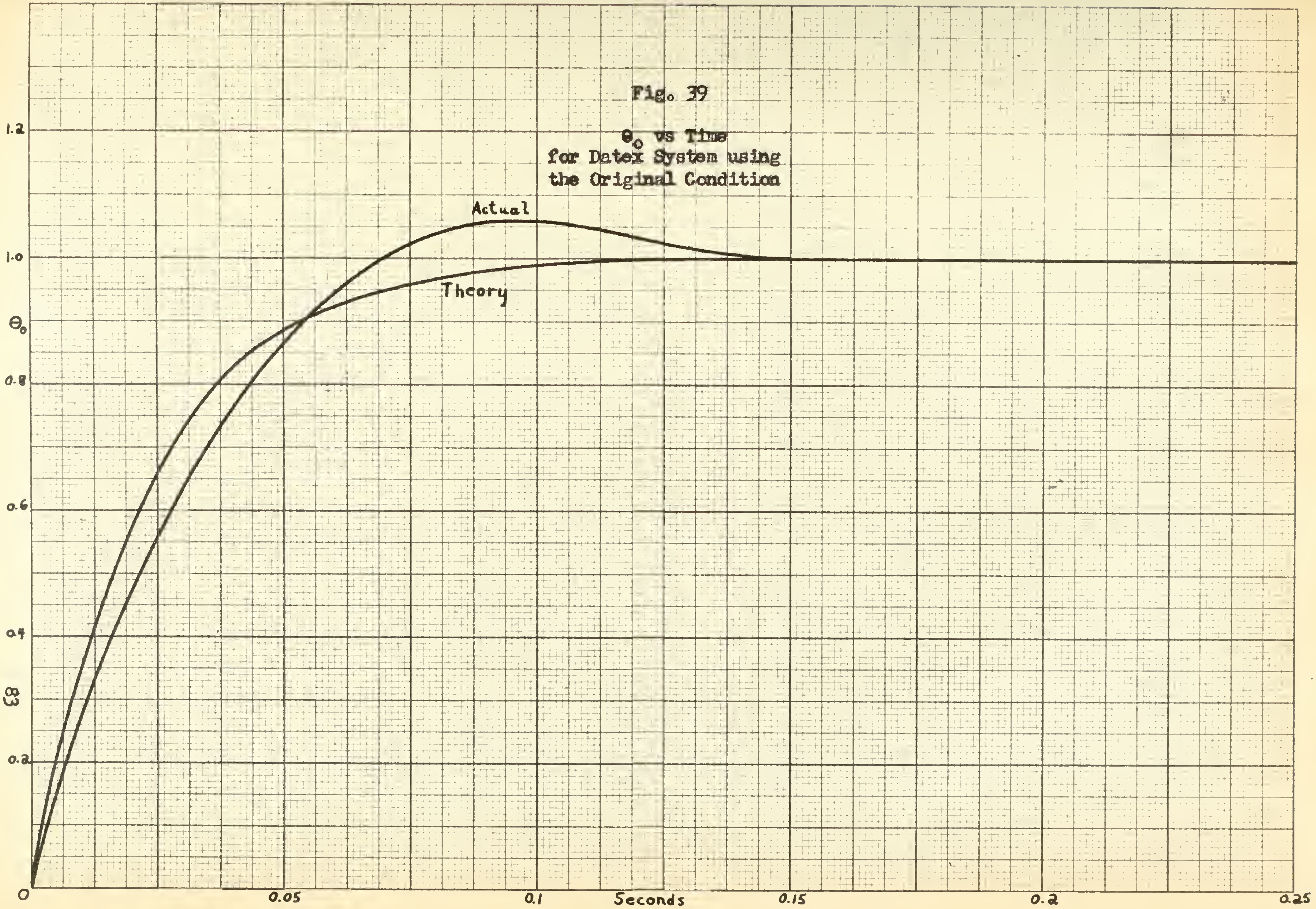


Fig. 39

Θ_o vs Time
for Datex System using
the Original Condition

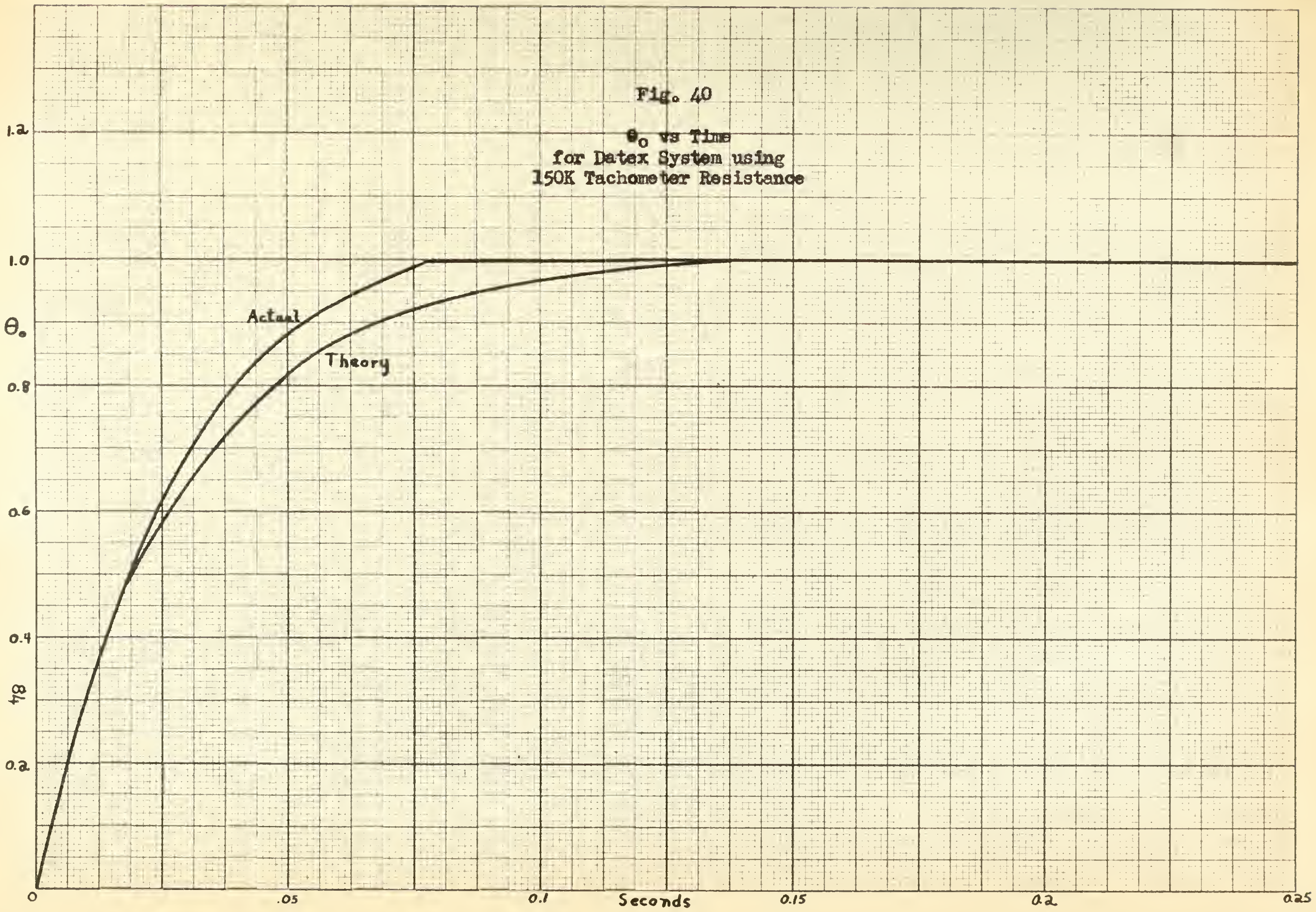


Fig. 40

θ_o vs Time
for Datex System using
150K Tachometer Resistance

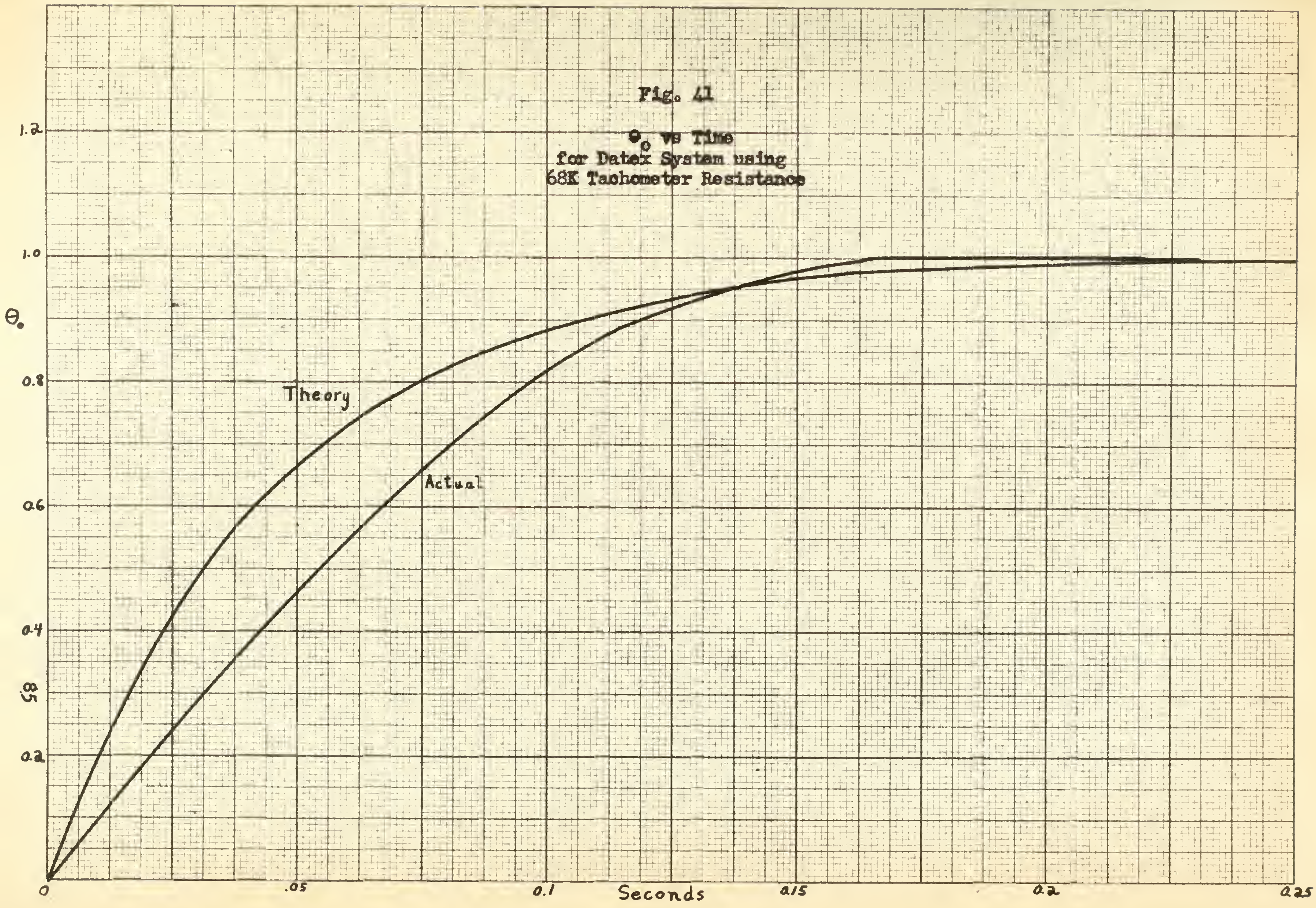


Fig. 41

Θ_o vs Time
for Datex System using
68K Tachometer Resistance

6. CONCLUSIONS AND COMPARISONS

The analysis of the Datex system appears to be confirmed by the actual transient response curves. The only major difference is the slight overshoot at the higher speeds. This is caused by the inertia of the encoder load, which becomes more prominent as the motor speed increases. The inertia was neglected in the analysis because it would have been divided by the gear ratio squared $(1800)^2$ and therefore would be very small. However, since the system is error limited, the actual transient response is present only in the small region less than 32 bits (5'16.4") and any value of the inertia will have a finite effect.

The inertia effect was more noticeable in the Baldwin system since its encoder inertia is much larger than that of the Datex encoder. The actual Baldwin transient response had a definite overshoot each time even at the high sampling rates because of this effect. However, the Baldwin encoder was designed to be installed in a large servo control system where any encoder inertia would be negligible.

The difference between the theoretical and actual Baldwin curves was also more noticeable because of the large amount of backlash in this system. A gear box with anti-backlash gears had been designed but its construction was not completed prior to the authors' departure from Philco, WDL. With the installation of the gear box and a correction for the inertia in the test system the Baldwin curves would probably coincide as closely as the Datex curves. Therefore, it can be concluded that the analysis is correct.

It would have been desirable, if time had permitted, to redesign and test each system as a type two. This would have enabled testing of the system with a known hydraulic servo motor that was intended for future

use at Philco. In addition, the motor speeds could have been increased and the tachometer feedback redesigned so that it would disconnect when the error limit was reached during a large angular input and would be reconnected as the limit was approached again. This would permit increased motor speeds in the error region greater than 32 bits and thus less time to reach the error null. The desirable feature of tachometer feedback would still be present when needed to reduce oscillations in the linear region less than 32 bits.

The two systems have the advantage of control simplicity and accuracy not possible in an ordinary analog system. This accuracy is determined mainly by the excellent manufacture of the two encoders. Speed of response can be easily changed by varying the tachometer feedback resistance. A potentiometer of the proper magnitude should be installed to enable the operator to obtain the best control performance.

Even though both encoders used in this work had discrimination of 2^{16} a really detailed comparison cannot be made since the optical type was based on sampling techniques while the mechanical type was continuous.

In a broad sense the Baldwin encoder could be called the "heavy duty" encoder while the Datex would fill the bill more satisfactorily in a lighter system. First of all the Baldwin encoder, due to its design, requires little or no gearing to operate with an input shaft, in fact one model has a center hole so that it can be mounted directly on a shaft. To employ it in a small rapidly moving system would require gearing that would introduce backlash and other non-linearities reducing accuracy. The opposite is true of the Datex. These encoders are built small and

are ideal for close, accurate, light-system application. The more rapid speed of response of the mechanical system would appear to be better employed on a system of relatively small mass, keeping down the number of oscillations, and magnitudes of torques.

If accuracy is the primary or sole consideration the obvious choice of these two encoders for a light system would be the Datex. However in a larger system it would be a choice between the reduced accuracy of an optical encoder with little or no gearing and the mechanical encoder whose system accuracy would gradually decrease as more gearing became necessary to couple it to the shaft.

In a system requiring time sharing, such as feeding the encoder outputs from three shafts (for instance azimuth, elevation, and transverse) to a computer, the optical encoder would be preferable with its variable sampling rate.

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APPENDIX A

Gray Binary Code

Conventional counting involves the use of the decimal system, in which ten different basic characters are used. The binary counting system, on the other hand, employs only two basic characters, and thus may be employed in electrical or optical counting systems in which only two basic conditions are recognized - "off and on".

If digital counting is to be done in binary and the result displayed in decimal, a simple relationship must be established. One of the simplest ways to do this is straight binary, as illustrated in Fig. A-1a. Here each vertical column, corresponding to a circular disc or ring on the encoder, represents a number equivalent to a power of two. If a horizontal pattern is read by a photocell or brush, a binary representation of position is obtained. This binary number can be interpreted in decimal numbers.

If readings of this pattern are taken with the photocells or brushes lying on a horizontal boundary line ambiguity of reading may result. This is illustrated by Fig. A-2a, where instead of reading 7 or 8 the misalignment or boundary reading may give 15 or 0 or some other combination. This effect is really the result of simultaneous changes of condition taking place in more than one vertical column or zone. The number of erroneous readings possible at a given boundary increases exponentially with the number of simultaneous zone changes.

Fig. A-1b illustrates a code devised to minimize ambiguous readings by allowing only one zone (the minimum) to change condition at a time. It is known as the "Gray" Binary Code in honor of its inventor,

Dr. Frank Gray, or, also, as the "Reflected" Binary Code. The maximum readout error from boundary line ambiguity when using this code cannot exceed the smallest reading increment, i.e., quantum. (A quantum is defined as a length equal to the least significant digit.) Even for gross misalignment the readout error will not be more than a few quanta (as illustrated in Fig. A-2b). This is a vast improvement over those possible by natural binary.

Binary Code	Binary Number 2^3	2^2	2^1	2^0	Decimal Equivalent
0					0
1				X	1
2			X		2
3			X	X	3
4		X			4
5		X		X	5
6		X	X		6
7		X	X	X	7
8	X				8
9	X			X	9
10	X		X		10

Fig. 1a

Natural Binary Code

Gray Code	Binary Number 2^3	2^2	2^1	2^0	Decimal Equivalent
0					0
1				X	1
2			X	X	3
3			X		2
4		X	X		6
5		X	X	X	7
6		X		X	5
7		X			4
8	X	X			12
9	X	X		X	13
10	X	X	X	X	15

Fig. 1b

Gray Binary Code

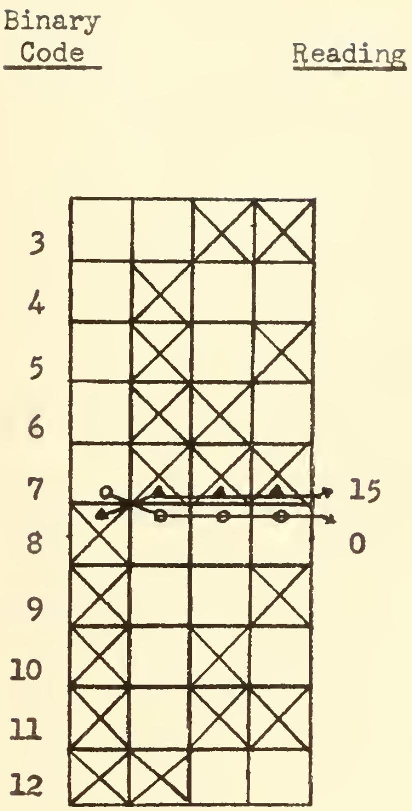


Fig. 2a
Ambiguity in Reading Natural
Binary Code

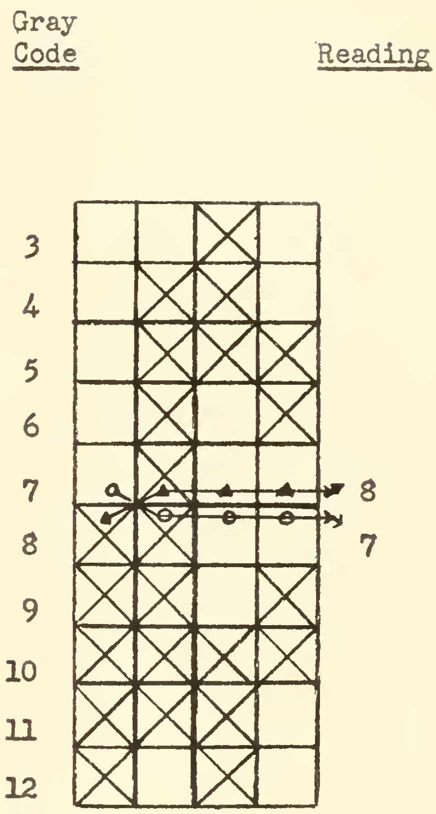


Fig. 2b
Ambiguity in Reading Gray
Binary Code

Key to Illustrations 1 through 5

A - Baldwin Encoder

B - Diehl A.C. Servo Motor

C - Input Switches; Sets A and B

D - Amplifier

E - Comparator

F - Translator

G - Baldwin Encoder Programmer

H - Reduction Gears, Baldwin System

I - Tachometer

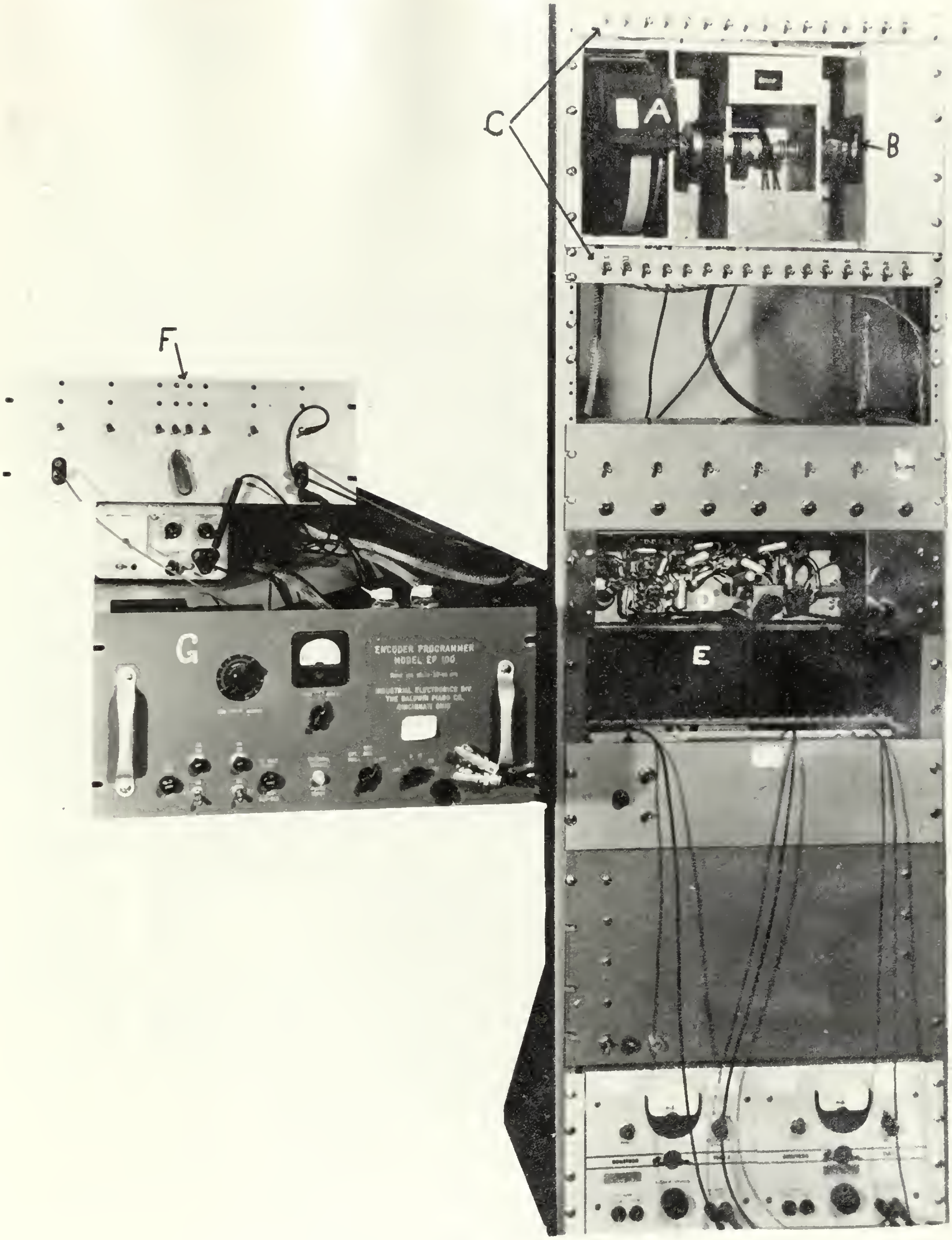
J - Reduction Gears, Datex System

K - Datex Encoder

L - Datex BA-701 Encoder Disc Selector

M - Relay Bank

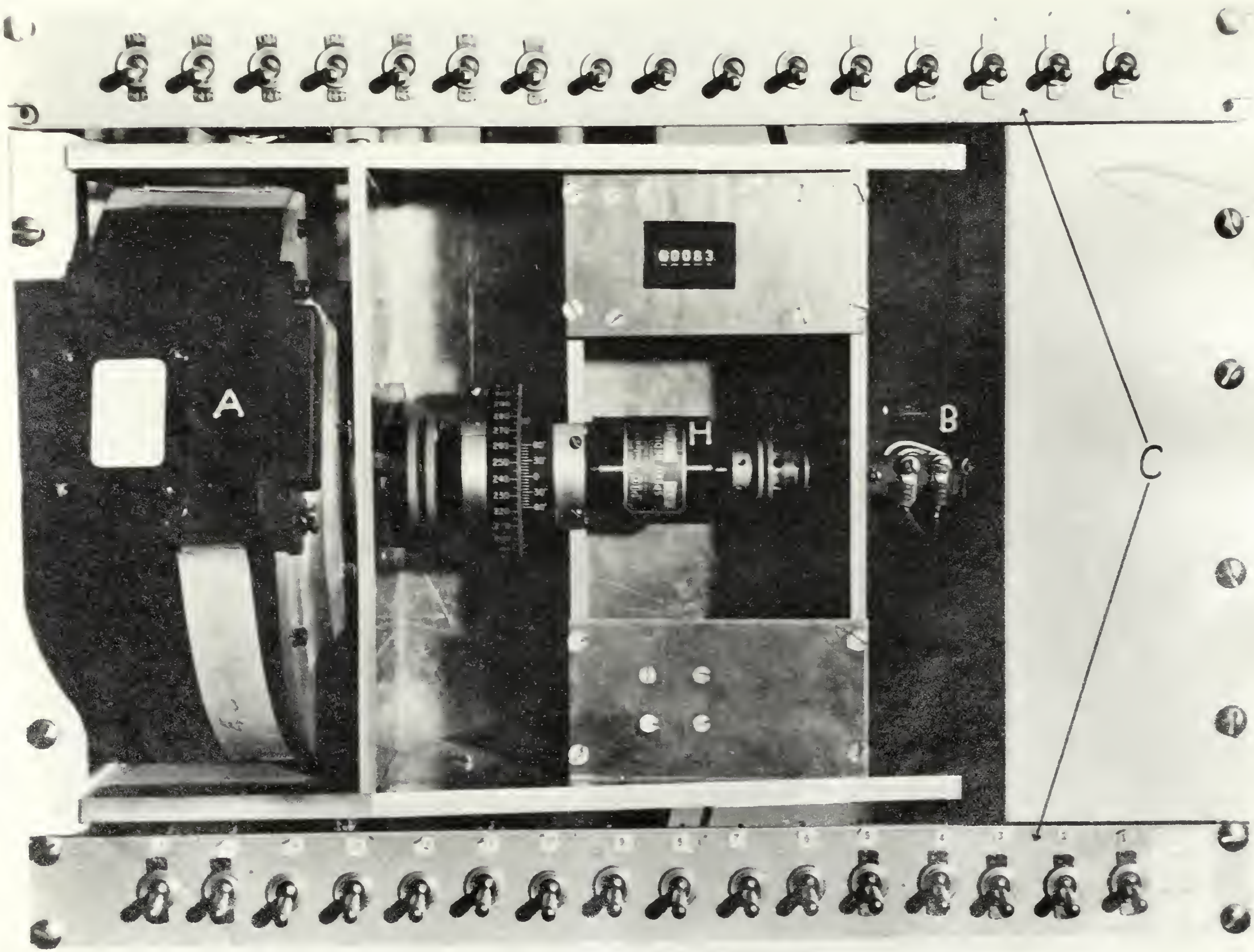
N - Typical Translator Board



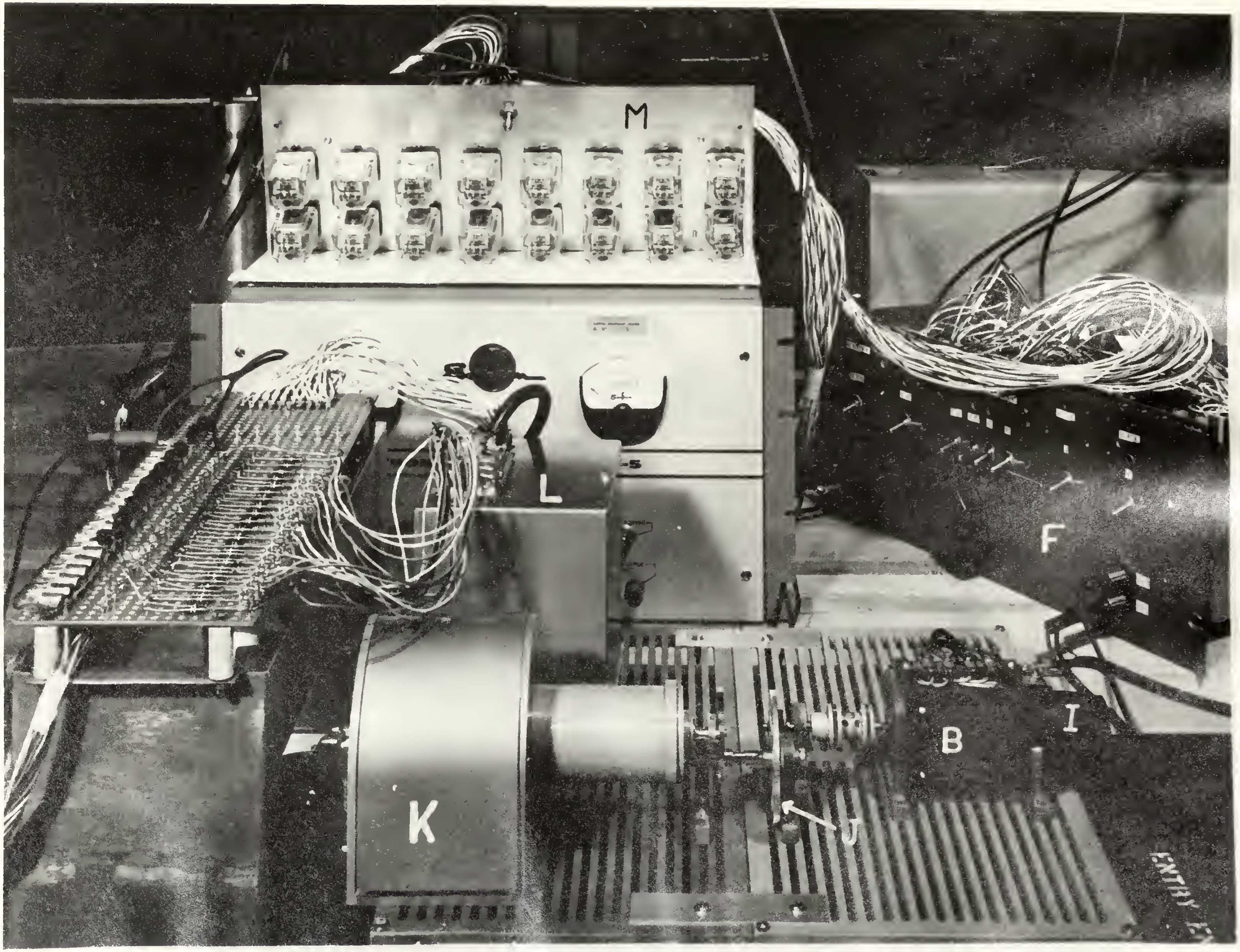
Illus. 1

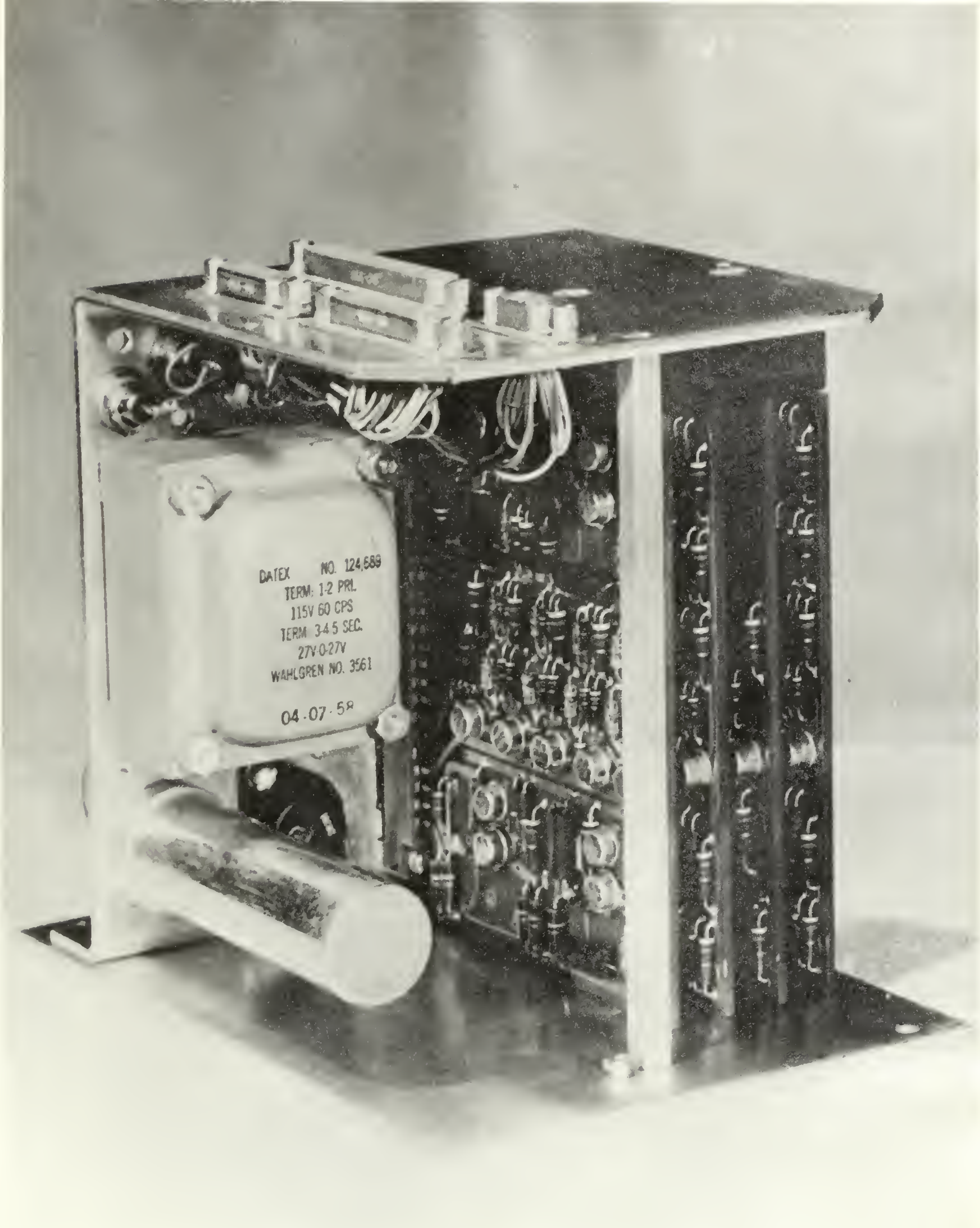
Servo Control System Equipment with Baldwin Encoder

Illus. 2 Baldwin Encoder and Servo Motor Connections



Illus. 3   Datex Encoder Connections and Switching Relay Bank




DATEX NO. 124,689
TERM: 1-2 PRI.
115V 60 CPS
TERM: 3-4-5 SEC.
27V-0-27V
WAHLGREN NO. 3561
04-07-58

Illus. 4

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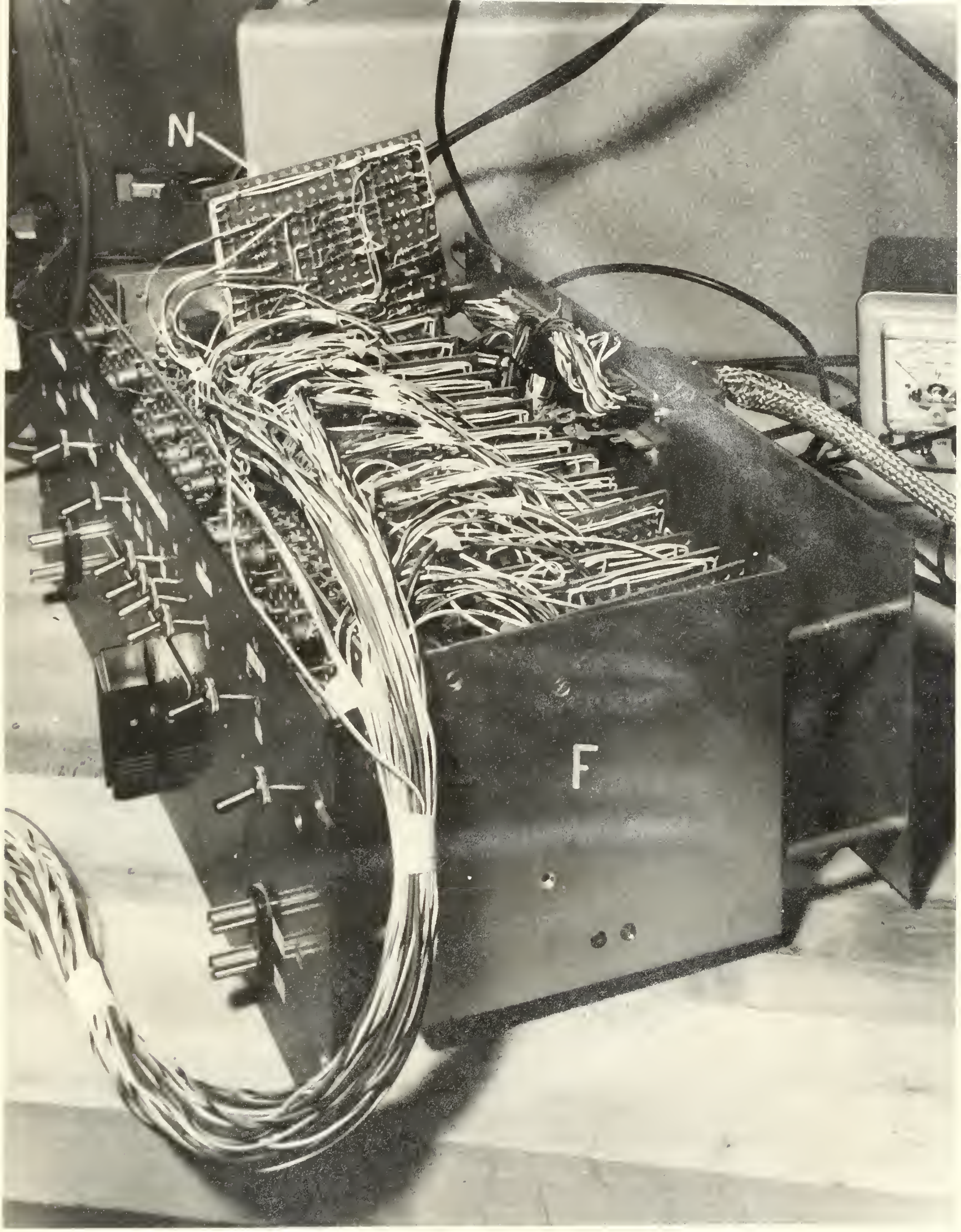
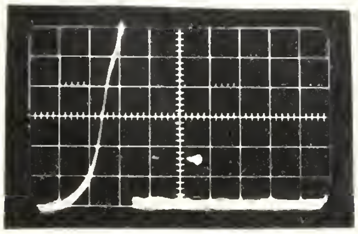


Illus. 5

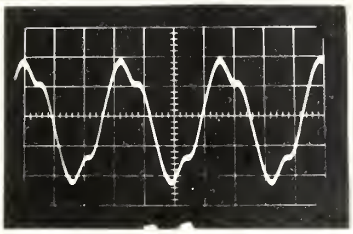
Translator



I[illegible]

[illegible]oder [illegible]tput

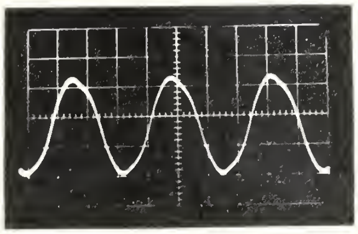
[illegible].5v/cm:1[illegible] μ[illegible]/c[illegible]



Illus. 6b

[illegible]mparator Output with no Input

0.[illegible]1v/cm; 5ms/cm



Illus. 6c

[illegible]omparator Output at Saturation

0.5v/cm; 5ms/cm